stat teaser

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FDS—A Power Tool for Designers of Optimization Experiments

We are devoting the majority of this issue to a white paper by Pat Whitcomb that details a major new statistical tool for evaluating alternative experiment designs. His article provides the pinnacle to a two-part series on power written by Stat-Ease Consultant Shari Kraber, which we published in the September and December 2007 issues of the Stat-Teaser—"No More Under-Sized Factorials" and "When Power is Too Low in Factorial Designs;" respectively. In a sidebar asking "Can I Use Power for Mixture and RSM?," Shari advised that "the power calculation is inappropriate for response surface (RSM) and mixture (MIX) design objectives, often reporting very low values." Shari promised "a future article on Fraction of Design Space (FDS) graphs which are a better tool to assess the capability of RSM and MIX designs." Here it is!

Fraction of Design Space (FDS)

When the goal is optimization (usually the case for RSM), the emphasis is on producing a fitted surface as precisely as possible. The response surface is drawn by predicting the mean outcome as a function of inputs over the region of experimentation. How precisely the surface can be drawn (or the mean values estimated) is a function of the standard error (SE) of the predicted mean response—the smaller the SE the better. The standard error of the predicted mean response at any point in the design space is a function of three things:

1. The experimental error (expressed as standard deviation).
2. The experiment design—the number of runs and their location.
3. Where the point is located in the design space (its coordinates).

Pat Whitcomb, President

Figure 1a (see page 2) shows a 3D plot of standard error for a two-factor face-centered (alpha = 1) central composite design (FCD). The red dots represent the coordinates of the design points, which range from minus 1 to plus 1 in coded factor units. (For two factors an FCD becomes equivalent to a full three-level factorial.) The upward (Z) axis displays the standard error of the predicted mean expressed as a relative value to the actual experimental error, which remains to be determined (think of this as a thought experiment!). In other words, when the standard deviation is known it simply becomes a multiplier on the Z-axis.

The shape of the surface depends on the

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Workshop Schedule

Experiment Design Made Easy
September 30–Oct. 2, 2008: Philadelphia, PA
November 4–6, 2008: Minneapolis, MN
December 9–11, 2008: Dallas, TX
Study the practical aspects of design of experiments (DOE). Learn about simple, but powerful, two-level factorial designs. $1495 ($1195 each, 3 or more)

Response Surface Methods for Process Optimization
September 23–25, 2008: Minneapolis, MN
Maximize profitability by discovering optimal process settings via RSM. $1495 ($1195 each, 3 or more)

Mixture Design for Optimal Formulations
October 21–23, 2008: Minneapolis, MN
Find the ideal recipes for your mixtures with high-powered statistical tools. $1495 ($1195 each, 3 or more)

DOE for DFSS: Variation by Design
November 11–12, 2008: Minneapolis, MN
Use DOE to create products and processes robust to varying conditions, and tolerance analysis to assure your specifications are met. A must for Design for Six Sigma (DFSS). $1195 ($995 each, 3 or more)

Designed Experiments for Life Sciences
November 18–20, 2008: Minneapolis, MN
Learn how to apply DOE to Life Science problems. $2050 ($1650 each, 3 or more)

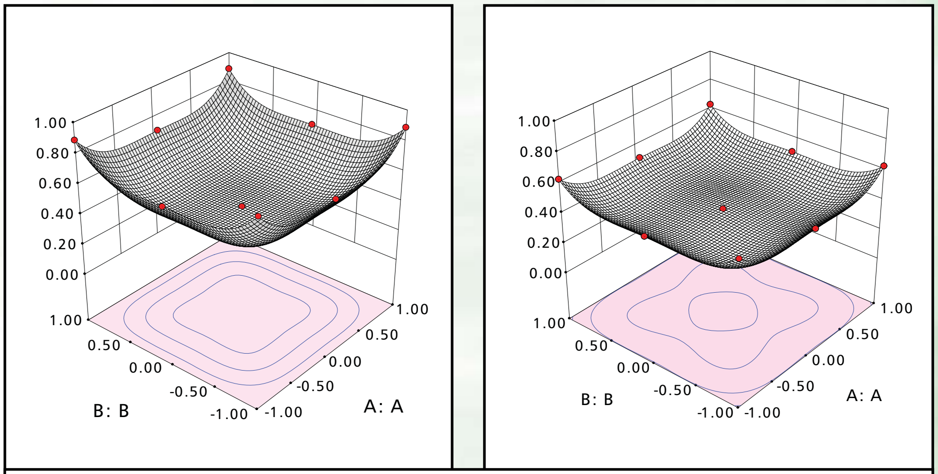
PreDOE: Basic Statistics for Experimenters (Web-Based)
PreDOE is an entry-level course for those who need to go back to the basics. See *http://www.statease.com/clas_pre.html* for more information. $95

Attendance is limited to 16. Contact Elicia* at 612.746.2038 or workshops@statease.com.

*See page 4 for a profile on Elicia and her white lab, Kaylee.

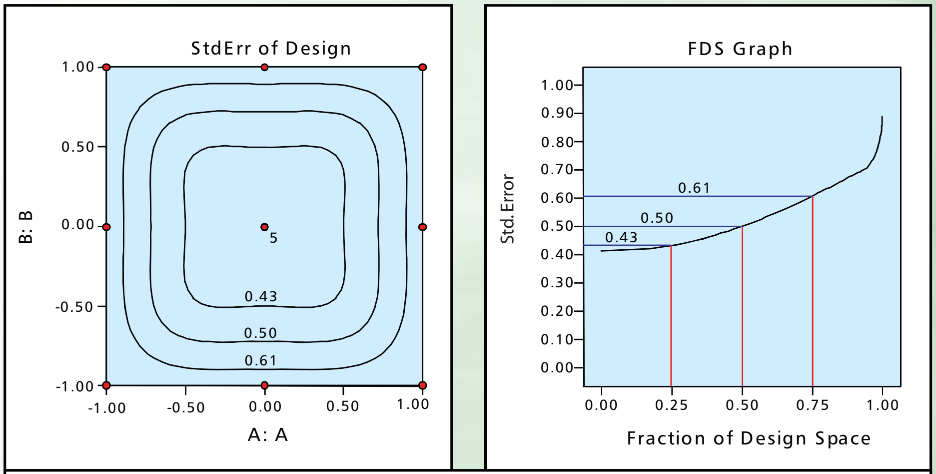
—*Continued from page 1*

design—the location of the run points and their number. For example, Figure 1b illustrates the impact on standard error from replication of the four vertices (corners) and four axial (edge) runs —the additional eight runs cause the SE surface to become flatter and lower. However, predictions around the perimeter of the design space still exhibit higher standard errors than near the center. This is not necessarily bad, assuming the experimenter centers the design at the most likely point for the potential optimum.



Figures 1a and 1b: 1a (left) represents an unreplicated face-centered design (FCD) and 1b (right) is the same design with 8 outer points replicated

A contour plot of the unreplicated FCD's standard error is shown in Figure 2a. This 2D representation will be easier to work with for numerical purposes. The three contours (0.43, 0.50 and 0.61) enclose 25%, 50% and 75% of the design space. In other words 0.25 fraction of the design space has a relative standard error less than 0.43, and so forth.



Figures 2a and 2b: 2a (left) shows a contour plot of an unreplicated FCD. In 2b (right) see the fraction of design space (FDS) plot for the FCD

The fraction of design space plot is shown in Figure 2b. It displays the area or volume of the design space having a mean standard error less than or equal to a specified value. (I've called out the SEs for FDSs of 0.25, 0.5 and 0.75 so you can see the connection to the contour plot.) The ratio of this volume to the total volume is the fraction of design space. Thus the FDS plot is a single plot showing the cumulative fraction of the design space on the x-axis (from zero to one) versus a standard error on the y-axis. It is a great tool to compare designs— look for lower (less error) and flatter (more uniform) profiles.

Sizing for Precision

For a given number of runs (dictated by the experimental budget), a design will ideally provide a fitted response surface that is precise throughout the region of interest. Statistically, precision is defined as the half-width (difference "d") of the confidence interval on the predicted mean value. Mathematically this is expressed as $\hat{y} \pm d$. As with any statistical interval, it depends on the degree of specified risk alpha (generally displayed by statisticians as the Greek symbol α, but we express it in English as "a"), which is typically set at 5% (0.05) to provide a confidence level of 95%.

To keep things really simple, let's work with a one-factor linear response surface experiment to illustrate how to size a design for precision. The solid center line in Figure 3 represents the predicted mean values for the fitted model.

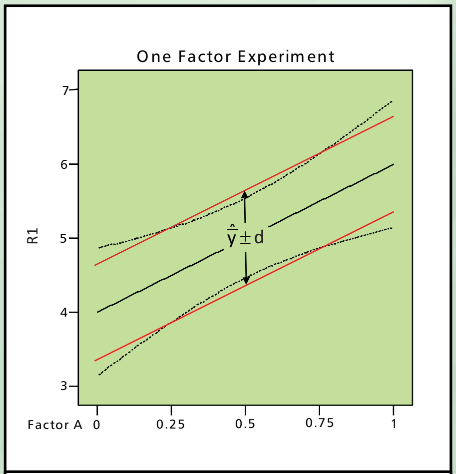


Figure 3: Linear fit with confidence limits displayed

The curved dotted lines are the com-

FDS—A Powerful Tool Continued...

puter-generated confidence limits, or the actual precision. Notice how they flare out at the extremes of the low and high levels of the factor (scaled from 0 to 1 in this case). The desired precision is shown by ±d, the half-width of the confidence interval, used to create the outer straight lines. As shown in the FCD example (Figures 1a, 1b, 2a and 2b), the precision obtained depends on where in the design space the prediction is made. In Figure 3 you see that less precision is obtained near the extremes. Precision in this case is best at the center (A = 0.5) but it remains adequate in the range from 0.25 to 0.75. However, the desired precision is not obtained for 0 to 0.25 or from 0.75 to 1. In other words, only about 50% of the design space meets the goal for precision. The FDS graph for this example is shown in Figure 4.

The legend for "reference Y" indicates that only 51% of the design space is precise enough to predict the mean within ±0.90 (entered for "d" in the floating FDS graph tool).

For those of you who would like to reproduce this one-factor example FDS graph in your Design-Expert® program (version 7.13 or later is needed), here are its parameters:

- Go to the "Response Surface" tab and choose the "One Factor" design.
- Set the factor range from 0 to 1.
- Change the "Model" from its default of "Quadratic" to "Linear" and build the design.
- The design should now be comprised of two runs at each extreme (0 and 1) plus one center point (0.5) for a total of 5 runs.
- Go to "Evaluation, Graphs" for FDS and enter a desired precision of ±0.90 (d = 0.90).
- Enter the overall standard deviation for the response, estimated to be 0.55 (s = 0.55).

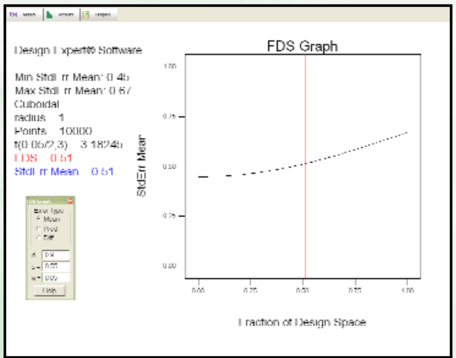


Figure 4: FDS graphs for one-factor response surface experiment

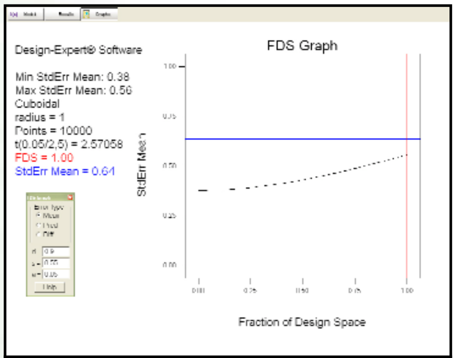


Figure 5: The FDS is greatly improved after adding only two more runs

FDS Guidelines

Typically, experimenters are trying to do as few runs as possible, and yet they desire high precision in predicting the response. The trick is to balance these missions when confronted with experimental error coming from variation in the process, in the sampling and in the response measurement (the testing). There are no hard and fast rules for what's required for the fraction of design space based on desired precision; but the following guidelines should help.

If you are mapping out an experimental region to explore for a significantly better response (optimization), be satisfied with a design that produces a FDS over 80 percent. However, if the experiment is the culmination of a series of developments and thus it must provide verification of manufacturing setup, then we advise you achieve an FDS above 95 percent.

Oops—the one-factor example shown above in Figure 4 falls short. What can be done to improve precision in a case like this? You could try to manage expectations by negotiating an increase in the difference "d," in other words be happy with less precise predictions. This might not very popular with your management and clients. Another tack might be to reduce the noise ("s"). Unfortunately this is likely to be systematic—a state of nature—and thus not amenable to improvement.

If you are willing to take on more risk, you could increase the alpha ("a")—that would help for FDS.

However, assuming you have some budget in reserve, perhaps the first thing to do is try adding more runs and see how much this improves your FDS. For example, in this case, by adding only two more runs at the two extremes, the FDS improves to 100 percent as shown in Figure 5. The augmented design obtains the desired precision throughout the entire design space. Life is good!

Conclusion

At the early stages of experimentation, screening and characterization, where factorials are the design of choice, the emphasis is on identifying main effects and interactions. For this purpose, power is an ideal metric to evaluate design suitability. However, when the goal is optimization (usually the case for RSM), the emphasis shifts to creating a fitted surface within a desired precision. Then fraction of design space (FDS) becomes a powerful tool for grading design suitability.

—**Pat Whitcomb, pat@statease.com**

Employee Profile — Elicia Bechard

09/08

Stat-Ease is pleased to introduce Elicia Bechard, Workshop Coordinator, who has been with us for a little more than a year. Her cheerful, sunny presence, as well as great organizational and interpersonal skills, have made her a welcome addition at Stat-Ease.

Education and Experience
Elicia is a graduate of the University of Minnesota, Morris with a BA degree in Psychology and Human Services, and a minor in French. Prior to coming to Stat-Ease her jobs included customer service, and 8 years at a local Methodist church as a Youth Director for 6th grade through college-age students.

Hobbies
Some of Elicia's favorite things include gardening, making jewelry and other crafts, reading books, cooking, and being an Auntie.

Elicia Bechard, Workshop Coordinator

Interesting Facts
Northern Minnesota has a large wilderness area called the Boundary Waters Canoe Area (BWCA). Elicia loves the outdoors and has been there nine times. She once jumped off a 30+ foot cliff—which cured her of the desire of ever doing it again!

She and her husband, Kevin, recently adopted a white lab puppy, Kaylee, in celebration of their first wedding anniversary. Kaylee has been very busy "training" them!

Kaylee at 2 months old

Workshop Tips
If you are interested in signing up for a Stat-Ease workshop, Elicia advises you to register early, communicate any dietary restrictions ahead of time, contact her for quantity discounts at *workshops@statease.com*, and don't forget to e-mail *stathelp@statease.com* after class if you have any questions!

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