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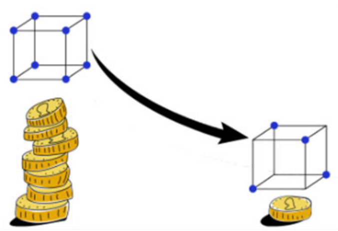
Reduce Experimental Runs with Fractional-Factorial Designs



Shari Kraber, MS Applied Statistics
Senior Client Success Manager
Stat-Ease, Inc.
shari@statease.com
Sales: sales@statease.com

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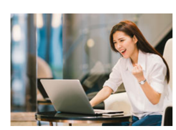
Maximizing this educational opportunity

Welcome everyone! To make the most from this webinar:

- Attendees on mute
- Questions addressed afterward
- Send further questions to shari@statease.com



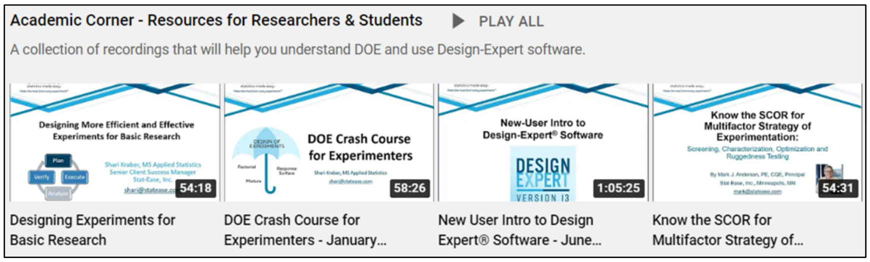
PS: Presentation posted to www.statease.com/webinars/

Please press the raise-hand button if you are with me.

Stat-Ease YouTube Channel

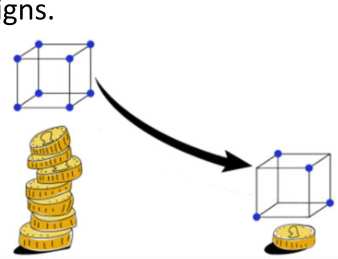
Key Reference: **StatisticsMadeEasybyStatEase**

On this channel, look for the **Academic Corner**:



Agenda: Fractional-Factorial Designs

- **Planning a DOE based on information objectives**
- Creating a fractional-factorial design
  - Concept of aliasing & why it matters
- Minimum-run designs
- Guide to choosing small-run designs.

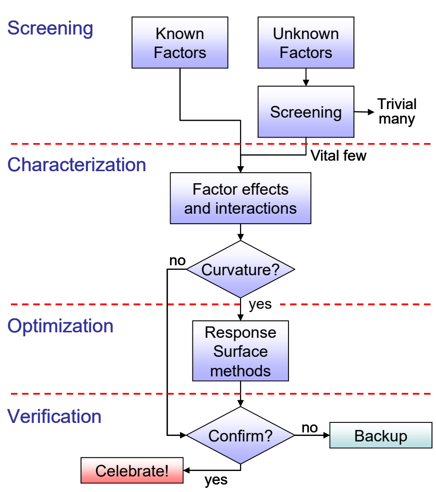


Planning the Experiment

1. Define the **response(s)** to be measured – make sure the measurement system will produce accurate and ideally, numerical results. (Not pass/fail)
2. What **factors** are likely to influence the response(s), and what factor range should be studied?
3. Choose a **design** that will provide the information you need. Consider your strategy of experimentation at this stage – are you doing:
   - Screening
   - Characterization
   - Optimization
   - Ruggedness Testing
4. Plan your budget in advance so you can afford to go back to the system to **run model verification points** to confirm your analysis results.

Plan: Strategy of Experimentation*

*YouTube: *Know the SCOR for Multifactor Strategy of Experimentation*



Design Objective

Detect main effects

↓

Define interactions

↓

Make precise predictions

Planning the Experiment
Choosing the Design based on Information

- Screening – Reveal the significant main (linear) effects in the system. Assuming that interactions exist in the system, use a design that keeps main effect information **unbiased** from two-factor interaction effects.
- Characterization – Identify both main effects and two-factor interactions. Use a design that estimates all two-factor interactions.
- Robustness/Ruggedness Testing – Confirm that a system is stable (no effects) over a very limited factor range. Use a design with a small number of runs (yet still meets power requirements).

Agenda: Fractional-Factorial Designs

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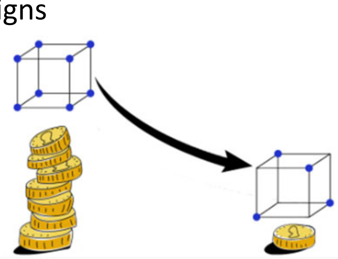

Fractional Factorial Design
Popcorn DOE as a 2^3: No Aliases

	A	B	C	Taste
1	–	–	–	74
2	+	–	–	75
3	–	+	–	71
4	+	+	–	80
5	–	–	+	81
6	+	–	+	77
7	–	+	+	42
8	+	+	+	32
Effect	-1.0	-20.5	-17.0	

Starting simple:

A typical 3-factor DOE with 8 runs. All columns of low/highs are unique. Effects are all calculated independently.

$$\text{Effect}(\Delta y) = \frac{\sum y_+}{n_+} - \frac{\sum y_-}{n_-}$$

Saving Runs with Fractional-Factorial Designs 9

Fractional Factorial Design
Popcorn DOE as a 2^3: All columns - No Aliases

	I	A	B	C	AB	AC	BC	ABC	Taste
1	+	–	–	–	+	+	+	–	74
2	+	+	–	–	–	–	+	+	75
3	+	–	+	–	–	+	–	+	71
4	+	+	+	–	+	–	–	–	80
5	+	–	–	+	+	–	–	+	81
6	+	+	–	+	–	+	–	–	77
7	+	–	+	+	–	–	+	–	42
8	+	+	+	+	+	+	+	+	32
	66.5	-1.0	-20.5	-17.0	0.5	-6.0	-21.5	-3.5	

To form a half fraction, eliminate the negative ABC rows!

Saving Runs with Fractional-Factorial Designs 10

Fractional Factorial Design
Popcorn as a 2^{3-1}: Aliases (Confounding)

	I	A	B	C	AB	AC	BC	ABC	Taste
1									
2	+	+	–	–	–	–	+	+	75
3	+	–	+	–	–	+	–	+	71
4									
5	+	–	–	+	+	–	–	+	81
6									
7									
8	+	+	+	+	+	+	+	+	32
	64.75	-22.5	-26.5	-16.5	-16.5	-26.5	-22.5	64.75	

What aliases have we created?

Fractional Factorial Design
2^{3-1} Aliases via Design-Expert Software

Factorial Effects Defining Contrast

I = ABC

[Intercept] = Intercept + ABC

[A] = A + BC

[B] = B + AC

[C] = C + AB

Resolution is determined by counting the letters in the shortest word in the defining contrast.

This is a Resolution III (**red**) design.

Main effects aliased with 2-factor interactions (2FI).

Aliases
What do they tell us?

2^3 Full		2^{3-1} Fraction	
A	−1.0	[A]	−22.5
B	−20.5	[B]	−26.5
C	−17.0	[C]	−16.5
AB	0.5		
AC	−6.0		
BC	−21.5		
ABC	−3.5		

Aliased effects are the linear combination of the true (unknown) effects.
In a Res **III** design, the main effects are *biased* by any existing two-factor interactions.

[A] = A + BC = (−1.0) + (−21.5) = −22.5

[B] = B + AC = (−20.5) + (−6.0) = −26.5

[C] = C + AB = (−17.0) + (0.5) = −16.5

4 Factors, 16 Runs (2^4 design)
All factor effects uniquely estimated

Std	I	A	B	C	D	AB	AC	AD	BC	BD	CD	ABC	ABD	ACD	BCD	ABCD	rate
1	+	−	−	−	−	+	+	+	+	+	+	−	−	−	−	+	45
2	+	+	−	−	−	−	−	−	+	+	+	+	+	+	−	−	71
3	+	−	+	−	−	−	+	+	−	−	+	+	+	−	+	−	48
4	+	+	+	−	−	+	−	−	−	−	+	−	−	+	+	+	65
5	+	−	−	+	−	+	−	+	−	+	−	+	−	+	+	−	68
6	+	+	−	+	−	−	+	−	−	+	−	−	+	−	+	+	60
7	+	−	+	+	−	−	−	+	+	−	−	−	+	+	−	+	80
8	+	+	+	+	−	+	+	−	+	−	−	+	−	−	−	−	65
9	+	−	−	−	+	+	+	−	+	−	−	−	+	+	+	−	43
10	+	+	−	−	+	−	−	+	+	−	−	+	−	−	+	+	100
11	+	−	+	−	+	−	+	−	−	+	−	+	−	+	−	+	45
12	+	+	+	−	+	+	−	+	−	+	−	−	+	−	−	−	104
13	+	−	−	+	+	+	−	−	−	−	+	+	+	−	−	+	75
14	+	+	−	+	+	−	+	+	−	−	+	−	−	+	−	−	86
15	+	−	+	+	+	−	−	−	+	+	+	−	−	−	+	−	70
16	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	96
	70.06	21.63	3.13	9.88	14.63	0.12	-18.12	16.63	2.38	-0.38	-1.13	1.88	4.13	-1.63	-2.62	1.37	

4 Factors, 8 Runs (2^{4-1} design)

Can you see the aliases?

Std	I	A	B	C	D	AB	AC	AD	BC	BD	CD	ABC	ABD	ACD	BCD	ABCD	rate
1	+	−	−	−	−	+	+	+	+	+	+	−	−	−	−	+	45
2																	
3																	
4	+	+	+	−	−	+	−	−	−	−	+	−	−	+	+	+	65
5																	
6	+	+	−	+	−	−	+	−	−	+	−	−	+	−	+	+	60
7	+	−	+	+	−	−	−	+	+	−	−	−	+	+	−	+	80
8																	
9																	
10	+	+	−	−	+	−	−	+	+	−	−	+	−	−	+	+	100
11	+	−	+	−	+	−	+	−	−	+	−	+	−	+	−	+	45
12																	
13	+	−	−	+	+	+	−	−	−	−	+	+	+	−	−	+	75
14																	
15																	
16	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	96
	70.8	19.0	1.5	14.0	16.5	-1.0	-18.5	19.0	19.0	-18.5	-1.0	16.5	14.0	1.5	19.0	70.8	

Fractional Factorial Design

2^{4-1} Aliases via Design-Expert Software

Factorial Effects Defining Contrast

I = ABCD

[Intercept] = Intercept + ABCD

[A] = A + BCD

[B] = B + ACD

[C] = C + ABD

[D] = D + ABC

Assumption: 3FI+ terms have negligible effects.

[AB] = AB + CD

[AC] = AC + BD

[AD] = AD + BC

This is a Resolution IV (yellow) design.

Main effects are estimated well. 2FI terms are aliased

Reactor Case Study

2^{5-1} Fractional Factorial

This is a Resolution V (**green**) design.

- Intercept aliased with a 5 FI,
- Main effects aliased with 4 FIs,
- 2 FIs aliased with 3 FIs.

Estimated Term	Aliased Terms
Intercept	= Intercept + ABCDE
A	= A + BCDE
B	= B + ACDE
C	= C + ABDE
D	= D + ABCE
E	= E + ABCD
AB	= AB + CDE
AC	= AC + BDE
AD	= AD + BCE
AE	= AE + BCD
BC	= BC + ADE
BD	= BD + ACE
BE	= BE + ACD
CD	= CD + ABE
CE	= CE + ABD
DE	= DE + ABC

Reactor Case Study

2^{5-1} Fractional Factorial

The Design-Expert selection matrix for two-level designs is color-coded:

- **Green** for go-ahead on resolution V or better,
- **Yellow** for proceed with caution on resolution IV,
- **Red** for stop on resolution III designs.

Number of Factors

Runs	2	3	4	5	6	7	8	9	10	11	12	13	14
4	2^{2}	2^{3-1}_{III}											
8		2^{3}	2^{4-1}_{IV}	2^{5-2}_{III}	2^{6-3}_{III}	2^{7-4}_{III}							
16			2^{4}	2^{5-1}_{V}	2^{6-2}_{IV}	2^{7-3}_{IV}	2^{8-4}_{IV}	2^{9-5}_{III}	2^{10-6}_{III}	2^{11-7}_{III}	2^{12-8}_{III}	2^{13-9}_{III}	2^{14-10}_{III}
32				2^{5}	2^{6-1}_{VI}	2^{7-2}_{IV}	2^{8-3}_{IV}	2^{9-4}_{IV}	2^{10-5}_{IV}	2^{11-6}_{IV}	2^{12-7}_{IV}	2^{13-8}_{IV}	2^{14-9}_{IV}
64					2^{6}	2^{7-1}_{VII}	2^{8-2}_{V}	2^{9-3}_{IV}	2^{10-4}_{IV}	2^{11-5}_{IV}	2^{12-6}_{IV}	2^{13-7}_{IV}	2^{14-8}_{IV}
128						2^{7}	2^{8-1}_{VIII}	2^{9-2}_{VI}	2^{10-3}_{V}	2^{11-4}_{V}	2^{12-5}_{IV}	2^{13-6}_{IV}	2^{14-7}_{IV}
256							2^{8}	2^{9-1}_{IX}	2^{10-2}_{VI}	2^{11-3}_{VI}	2^{12-4}_{VI}	2^{13-5}_{V}	2^{14-6}_{V}

Fractional Factorial Design
Exploring Alias Structures

Let's look at:

2 levels, 7 factors,
1/8th fraction = 16 runs

- $2^{7\text{-}3}$, 7 factors in 16 runs:
  Solid Res IV: All 21 two-factor interactions aliased with each other.
- $2^{7\text{-}2}$, 7 factors in 32 runs:
  Barely Res IV: Most (15) 2FIs aliased with 3FIs only, i.e., cleanly.

Resolution only tells the worst that can happen.
***Always** examine the alias structure when setting up and analyzing your designs.*

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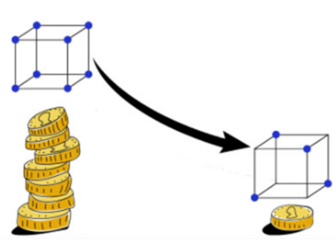


Minimum Run Characterize (Res V) (MR5) Designs*

Regular fractions (2^{k-p} fractional factorials) of 2^k designs often contain more runs than necessary to estimate the coefficients in the 2FI model.

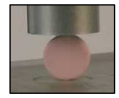
- The smallest regular resolution V design for k=7 uses 64 runs (2^{7-1}) to estimate 29 coefficients.
- Our balanced minimum run resolution V (MR5) design for k=7 has 30 runs, a savings of 34 runs.
- Disadvantage – partial aliasing. MR5 designs are irregular fractions, so effect estimates are dependent on other terms chosen.

Minimum Run Characterize (Res V) Designs
Provide Considerable Savings

k	2^{k-p}	MR5
6	32	22
7	64	30
8	64	38
9	128	46
10	128	56
11	128	68
12	256	80
13	256	92
14	256	106

k	2^{k-p}	MR5
15	256	122
16	256	138
17	256	154
18	512	172
19	512	192
20	512	212
21	512	232
25	1024	326
30	1024	466

Tablet Strength MR5 Design

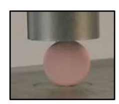


A process development team wants to characterize the effects (estimate all MEs and 2FIs)of eight factors on the hardness and friability of their tablets. Design choices include:

- 64 run 2^{8-2} resolution V fraction.
- 38 run Minimum-Run resolution V (MR5) design.

The teams decides to use the 38 run MR5 design.

Tablet Strength MR5 Design
Background - Factors

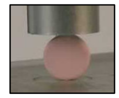


There are eight factors. The first three are properties of the powder and the last five are machine settings.

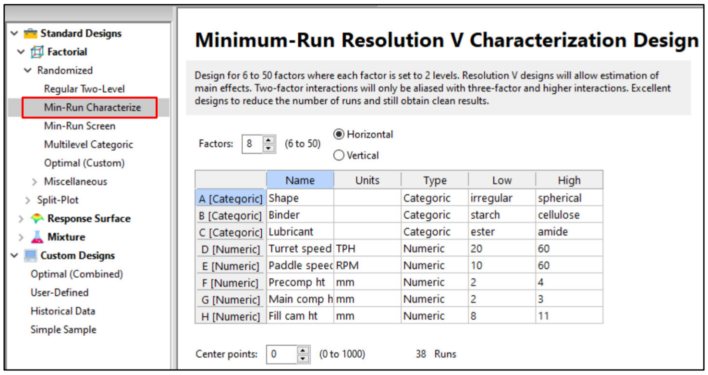
Factor	Name	Units
A	Shape	
B	Binder	
C	Lubricant	
D	Turret speed	THP
E	Feed frame paddle speed	RPM
F	Tablet cylinder height, precomp	mm
G	Tablet cylinder height, main compress	mm
H	Fill cam height	mm

Tablet Strength MR5 Design

Build Design *(page 1 of 3)*



1. Build the design:



Minimum-Run Resolution V Characterization Design

Design for 6 to 50 factors where each factor is set to 2 levels. Resolution V designs will allow estimation of main effects. Two-factor interactions will only be aliased with three-factor and higher interactions. Excellent designs to reduce the number of runs and still obtain clean results.

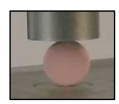
Factors: 8 (6 to 50) ◉ Horizontal ○ Vertical

	Name	Units	Type	Low	High
A [Categoric]	Shape		Categoric	irregular	spherical
B [Categoric]	Binder		Categoric	starch	cellulose
C [Categoric]	Lubricant		Categoric	ester	amide
D [Numeric]	Turret speed	TPH	Numeric	20	60
E [Numeric]	Paddle speec	RPM	Numeric	10	60
F [Numeric]	Precomp ht	mm	Numeric	2	4
G [Numeric]	Main comp h	mm	Numeric	2	3
H [Numeric]	Fill cam ht	mm	Numeric	8	11

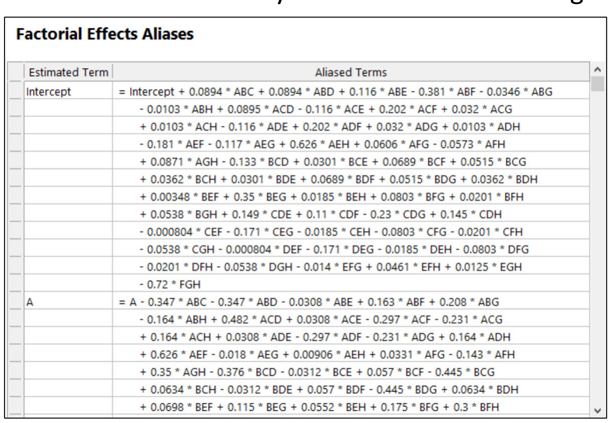
Center points: 0 (0 to 1000) 38 Runs

Tablet Strength MR5 Design

Build Design *(page 2 of 3)*



2. Note that MEs and 2FIs are only aliased with 3FIs and higher:

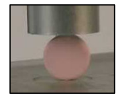


Factorial Effects Aliases

Estimated Term	Aliased Terms
Intercept	= Intercept + 0.0894 * ABC + 0.0894 * ABD + 0.116 * ABE - 0.381 * ABF - 0.0346 * ABG
	- 0.0103 * ABH + 0.0895 * ACD - 0.116 * ACE + 0.202 * ACF + 0.032 * ACG
	+ 0.0103 * ACH - 0.116 * ADE + 0.202 * ADF + 0.032 * ADG + 0.0103 * ADH
	- 0.181 * AEF - 0.117 * AEG + 0.626 * AEH + 0.0606 * AFG - 0.0573 * AFH
	+ 0.0871 * AGH - 0.133 * BCD + 0.0301 * BCE + 0.0689 * BCF + 0.0515 * BCG
	+ 0.0362 * BCH + 0.0301 * BDE + 0.0689 * BDF + 0.0515 * BDG + 0.0362 * BDH
	+ 0.00348 * BEF + 0.35 * BEG + 0.0185 * BEH + 0.0803 * BFG + 0.0201 * BFH
	+ 0.0538 * BGH + 0.149 * CDE + 0.11 * CDF - 0.23 * CDG + 0.145 * CDH
	- 0.000804 * CEF - 0.171 * CEG - 0.0185 * CEH - 0.0803 * CFG - 0.0201 * CFH
	- 0.0538 * CGH - 0.000804 * DEF - 0.171 * DEG - 0.0185 * DEH - 0.0803 * DFG
	- 0.0201 * DFH - 0.0538 * DGH - 0.014 * EFG + 0.0461 * EFH + 0.0125 * EGH
	- 0.72 * FGH
A	= A - 0.347 * ABC - 0.347 * ABD - 0.0308 * ABE + 0.163 * ABF + 0.208 * ABG
	- 0.164 * ABH + 0.482 * ACD + 0.0308 * ACE - 0.297 * ACF - 0.231 * ACG
	+ 0.164 * ACH + 0.0308 * ADE - 0.297 * ADF - 0.231 * ADG + 0.164 * ADH
	+ 0.626 * AEF - 0.018 * AEG + 0.00906 * AEH + 0.0331 * AFG - 0.143 * AFH
	+ 0.35 * AGH - 0.376 * BCD - 0.0312 * BCE + 0.057 * BCF - 0.445 * BCG
	+ 0.0634 * BCH - 0.0312 * BDE + 0.057 * BDF - 0.445 * BDG + 0.0634 * BDH
	+ 0.0698 * BEF + 0.115 * BEG + 0.0552 * BEH + 0.175 * BFG + 0.3 * BFH

Tablet Strength MR5 Design

Build Design *(page 3 of 3)*



3. There are two responses:

Responses: 2 (1 to 999) ◉ Horizontal ○ Vertical [Edit Model...] ☐ Edit response types

Name	Units	Diff. to detect Delta("Signal")	Est. Std. Dev. Sigma("Noise")	Delta/Sigma (Signal/Noise Ratio)
Hardness	kg	0.5	0.5	1
Friablilty	%	0.1	0.1	1

Next >>

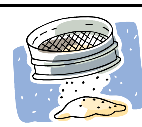
4. Power is adequate:

Name	Units	Delta (Signal)	Sigma (Noise)	Signal/ Noise	Power for A	Power for B	Power for C	Power for D	Power for E	Power for F	Power for G	Power for H
Hardness	kg	0.5	0.5	1	84.0%	82.8%	81.1%	81.1%	83.9%	84.0%	84.0%	83.9%
Friablilty	%	0.1	0.1	1	84.0%	82.8%	81.1%	81.1%	83.9%	84.0%	84.0%	83.9%

Finish

Minimum-Run Screening

MR4 Designs*



The concept of screening is to discover the vital few primary factors that drive the process. We assume that interactions may exist in the system.

Minimum-Run Screening (MR4):

- MR4 designs are for absolute-minimum-run screening.
- They often offer considerable savings versus a standard 2^{k-p} fraction with the same resolution.
- They require only two runs for each factor (i.e., runs = 2k).
- However, for robust design, we advise that you go with the "minimum runs plus 2" option.

See the next slide for details. ☞

Minimum-Run Screening
(MR4+2) Designs

Problems:

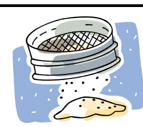
- If even 1 run lost, design becomes resolution III – main effects become badly aliased.
- Reduction in runs causes power loss – may miss significant effects.

Evaluate power before doing experiment.

Solution:

- To risks of resolution loss and to increase power, add some padding: Use the **MR4+2** designs (DX default).

MR4 (+2) Designs
Provide Considerable Savings



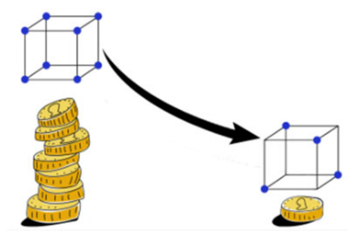
k	2^{k-p}	MR4+2
5	16	12
6	16	14
7	16	16
8	16*	18
9	32	20
10	32	22
11	32	24
12	32	26
13	32	28
14	32	30
15	32	32

k	2^{k-p}	MR4+2
16	32*	34
17	64	36
19	64	40
20	64	42
21	64	44
25	64	52
30	64	62
35	128	72
40	128	82
45	128	92
50	128	102

** No savings for 8, 16 (or 32) factors.*

Agenda: Fractional-Factorial Designs

- Planning a DOE based on information objectives
- Creating a fractional-factorial design
  - Concept of aliasing & why it matters
- Minimum-run designs
- **Guide to choosing small-run designs**



Planning the Experiment
Choosing the Design

- Screening – assuming that interactions exist in the system, use a design that keeps main effect information **unbiased** from two-factor interaction effects.
  - Resolution IV (yellow), Res V (green) or better designs
  - NOT Res III (red) Plackett-Burman or Taguchi designs
- Characterization – use a design that can estimate all two-factor interactions (Full factorial or Resolution V+).
  - Consider adding center points to test for non-linearity (curvature)
- Robustness/Ruggedness Testing – use a design with a small number of runs (yet still meets power requirements) (Resolution III or IV).

Guide to Using Small-Run Designs
Screening

Goal: Provide unbiased estimates of the main effects; i.e., not confounded by two-factor interactions.

Screening designs:

- 2^{k-p} algebraic fractions of resolution IV (or higher)
- Min-Run Screen (MR4)
  (should be padded by 2 extra runs, MR4+2)
- Definitive Screening Designs (DSDs)*
  (in Response Surface section – use RSM analysis)

Warning: *In the presence of two-factor interactions resolution III designs will give misleading information.*

Guide to Using Small-Run Designs
Characterization

Goal: Provide estimates of main effects and two-factor interactions.

Characterization designs:

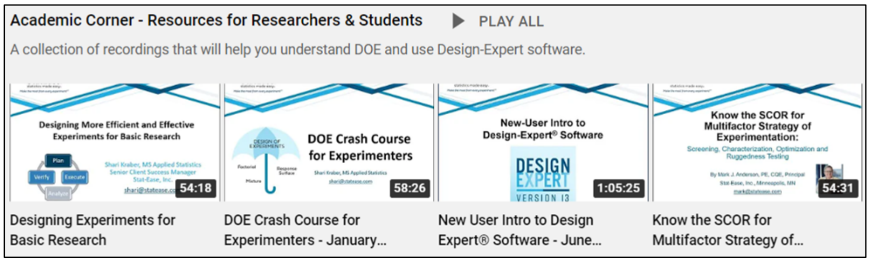
- Full factorials
- 2^{k-p} algebraic fractions of resolution V (or higher)
- Min-Run Characterize (MR5)

Note: ***Do not replicate a fractional factorial to increase power.*** *Add a new fraction instead. This increases power, reduces aliases and adds new combinations to better fill the DOE space.*

Stat-Ease YouTube Channel

Key Reference: **StatisticsMadeEasybyStatEase**

On this channel, look for the **Academic Corner**:



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