Strategies for Sequential Experimentation

Martin Bezener, PhD
Stat-Ease, Inc.

martin@statease.com

Sequential DOE

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Outline

- Introduction and Motivation
- Sequential DOE Strategies
- An Example
- Conclusion

Sequential DOE 2

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Outline

- **Introduction and Motivation**
- Sequential DOE Strategies
- An Example
- Conclusion

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Introduction

Traditional Experimentation

- Traditional Experimentation is often taught as a **one-shot** approach.
- Here is a typical textbook example:
  - Choose **factors** and define the low/high values of each
  - Choose an appropriate experimental design
  - Compute the expected **power** (or precision) of the experimental design
  - Add or subtract runs needed to achieve the target **power** (or precision)
  - Perform experiment and analyze data
  - Optimize to find factor settings that achieve experimental targets
  - Confirm the results of the analysis

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Introduction

Zoom In

- Most textbooks offer no practical advice on these two steps:
  - Compute the expected **power** (or precision) of the experimental design
  - Add or subtract runs needed to achieve the target **power** (or precision)
- In my experience, experimenters usually **do not** have an unlimited budget of runs available. They usually must settle for power or precision lower than "what the textbook says" which is often around 80%.
- This usually means that whatever run budget is available will be spent in a single execution of the experiment.
- In my experience, this is often wasteful, and experimenters sell themselves short because experimentation is taught as a "one-shot" approach.

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Real Life DOE

The One-Shot Approach

- Here's a situation I run into all the time.
- Suppose you have a **3**-component mixture experiment and a run budget of **14** runs.
- The component ranges are
  - $0.2 \le \mathbf{x}_1 \le 0.6$
  - $0.2 \le \mathbf{x}_2 \le 0.6$
  - $0.2 \le \mathbf{x}_3 \le 0.6$ $\qquad \mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3 = 1.0$
- An experimenter, following standard textbook advice, would likely build an optimal design with 14 runs that may look something like…

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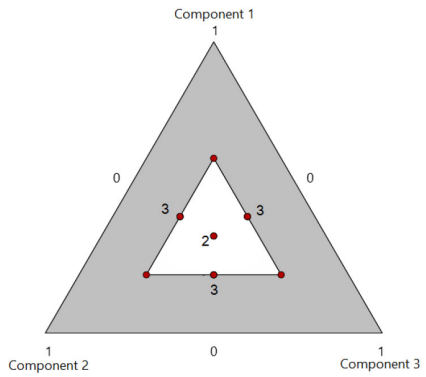
Real Life DOE

14-Run Optimal Experimental Design



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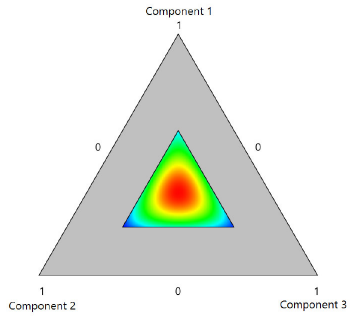
Real Life DOE

What's the Problem??

- In theory, there is no problem. Because in textbooks the final result of the analysis always looks like this:



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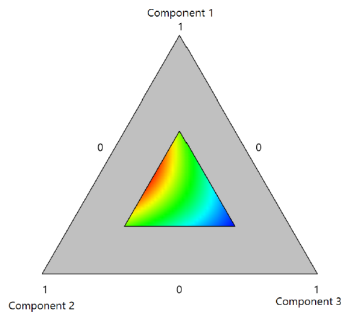
Real Life DOE

What Usually Happens

- More often that not, this is what happens in real problems:



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Real Life DOE

What Now?

- We had a run budget of 14 runs. We performed a 14-run experiment.
- The results show that we may not have fully optimized our process, because the maximum occurs at an edge of the design space.
- So where do we go now?
  - Accept the results as “good enough” and move on?
  - Extrapolate outside of the original design space?
  - Throw away the data and start over?
- Could we have done **something else** to avoid this situation?

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Outline

- Introduction and Motivation
- **Sequential DOE Strategies**
- An Example
- Conclusion

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Design Space Strategies

An Alternative Approach

- Instead of using the entire run budget to do a one-shot experiment, why not use 75% of the run budget on a first experiment, and then allocate the remaining 25% of the run budget based on the results of the first pass?
- In the previous example, we could do a **10**-run experiment first to get a sense of what's going on.
- Based on the results of the first **10**-run experiment, we would then allocate the remaining **4** runs.
- If an optimum occurs along an edge, we can reshape the design space before allocating our remaining **4** runs to fill in the additional space.
- If the optimum occurs inside of the original design space, we can allocate the remaining **4** runs within the original design space.

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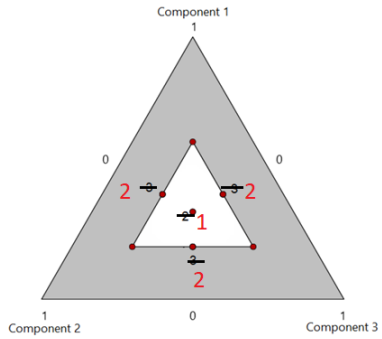
Design Space Strategies

An Alternative Approach

- Start with a 10-run design:



Sequential DOE 13

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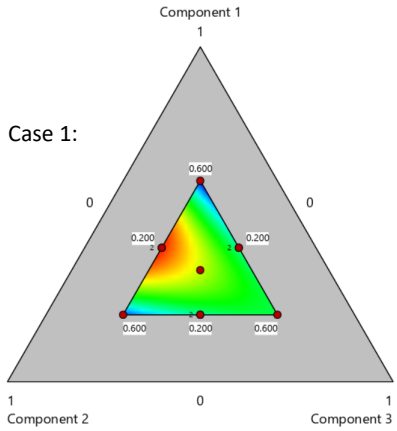
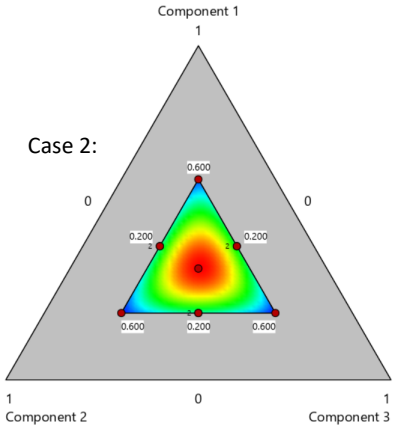
An Alternative Approach

Two Things Can Happen

Case 1:



Case 2:



Sequential DOE 14

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The Next Step

Case 1: Optimum at an Edge or Vertex

- In this case, the optimum lies along an edge. It's unclear whether this region is the true optimum – perhaps the process can be further optimized.
- Perhaps the red region is "good enough" and we can stop there.
- In my experience, many experimenters will either stop because the run budget has been depleted, or throw away this data and start over.
- We performed 10 runs – we still have a budget of 4 runs left! Is there a way to solve this problem without having to start over or settle for the sub-par results?
- Yes! We can **augment** the original design space and perform the remaining 4 runs in the larger experimental design space!

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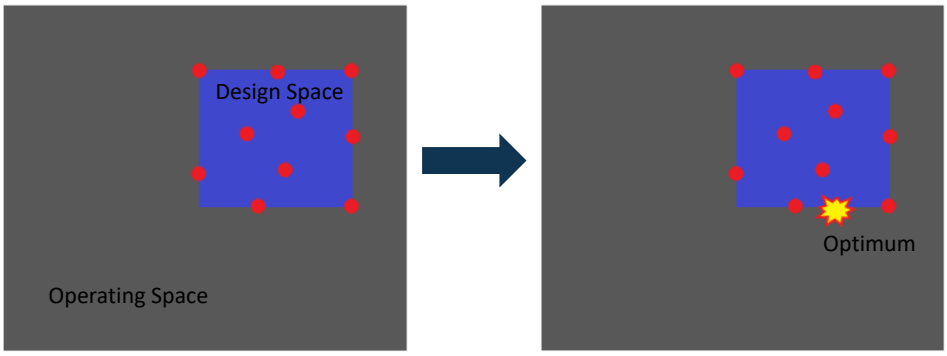
The Next Step

The Augmented Design Space



The optimum occurs at the edge of the design space, so we expand the space.

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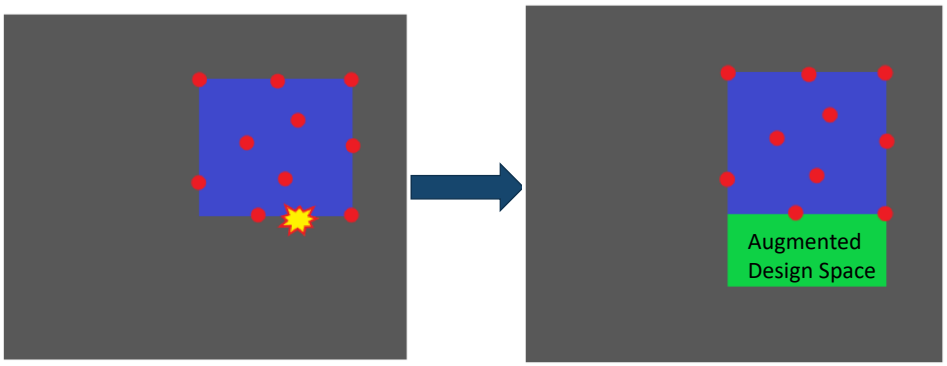
The Next Step

The Augmented Design Space



The green space is added to the original design space to try to capture the optimum.

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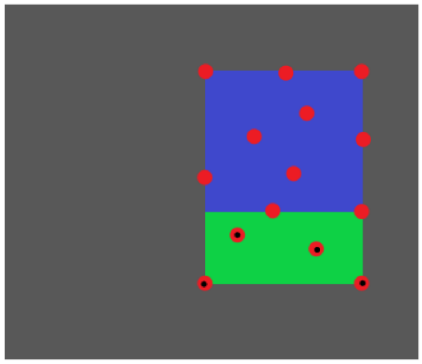
The Next Step

The Augmented Design Space

- After we augment our original design space, we add additional runs into the larger design space to try to capture the optimum.



You don't need to throw away the original data!

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Take Home Message

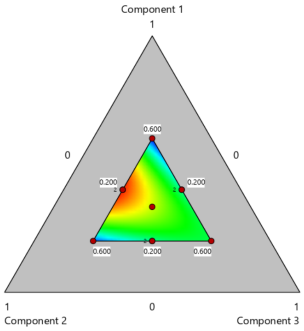
Some Key Points:

- If we use our entire run budget in the original design space, then we cannot adapt to unexpected results, such as optima at the extremes.
- By keeping some runs in reserve, we can change the shape of the experimental design space as needed.
- You can sequentially expand the space several times as needed!

Mixture Example

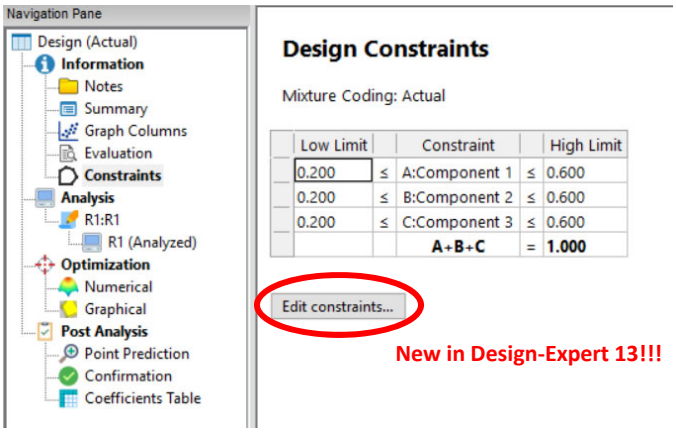
Returning to the Mixture Example

- Recall the 10-run mixture example (14 run budget) with the following result. How can we use the Design-Expert software to experiment sequentially?



Use the Software!

How to Expand the Design Space?



New in Design-Expert 13!!!

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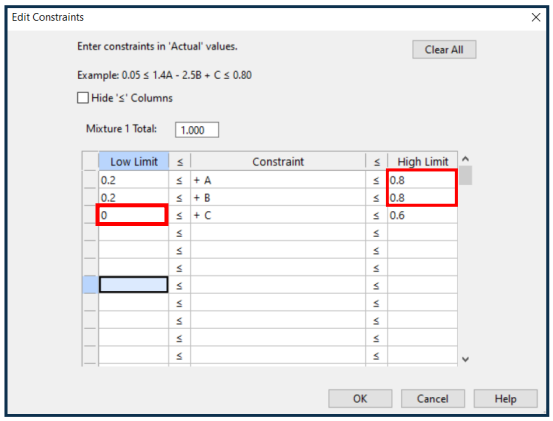
Use the Software!

Enter the New Ranges:



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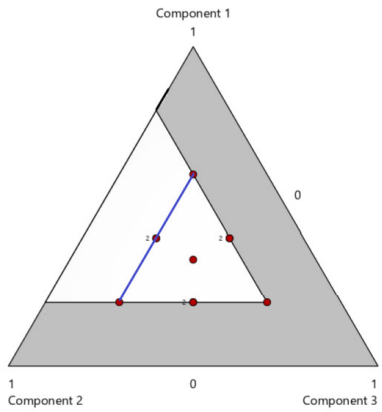
Use the Software!

The Expanded Design Space



Now we have expanded the ranges of two components. We need to add some additional runs to fill the space.

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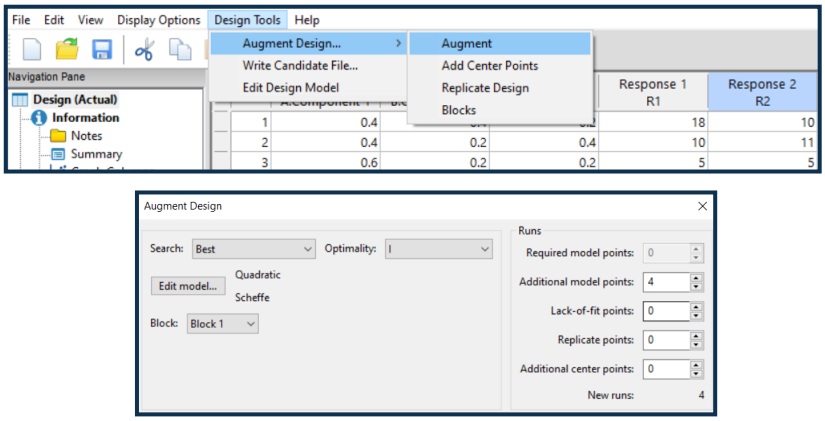
Use the Software!

Now Fill the Augmented Space



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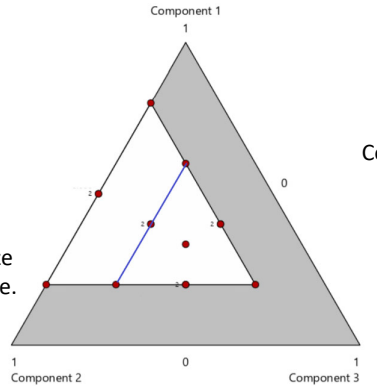
Use the Software!

The Augmented Experimental Design



Consider Blocking!

To Finish: perform an additional 4 runs, analyze the data, and re-optimize.

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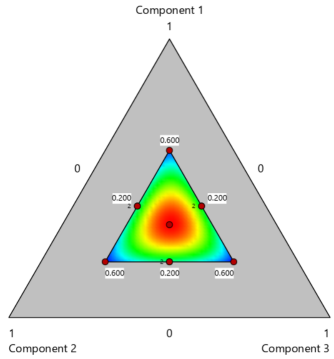
Another Case

What if Everything is Okay?

- Remember, there **is** a chance that everything goes right!



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The Next Step

What's Next?

- Let's say you do only 75% of your run budget and everything goes well, like in the picture on the previous slide.
- The question then becomes: what to do if you don't need to augment the original design space?
- How do you allocate the remaining runs?
- **Strategy:** allocate the remaining runs in a region around the optimum of the first experiment. Don't allocate the runs as if though you had no prior information.
- **Why?** Adding additional runs near a potential optimum will improve the precision of the design in the area where it matters most.

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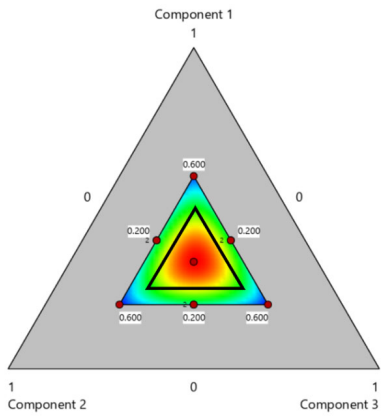
The Next Step

The New Region



Notice the triangle inscribed within the original space. We'll put the 4 additional points inside of that triangle.

The black triangle is defined as:
$0.25 \le \mathbf{x}_1 \le 0.50$
$0.25 \le \mathbf{x}_2 \le 0.50$
$0.25 \le \mathbf{x}3 \le 0.50$

Sequential DOE 28

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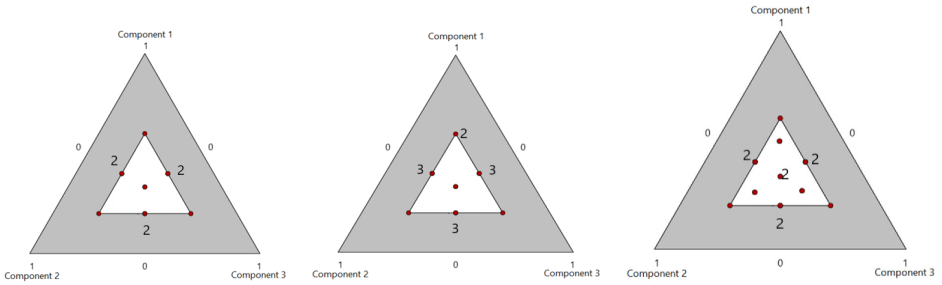
The Next Step

Compare Approaches



Original Design	4 Runs in Original Space	4 Runs in Region of Interest
SE = 0.50	SE = 0.43	SE = 0.40

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- Introduction and Motivation
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- **An Example**
- Conclusion

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An Example

Sunscreen

- A 7-component mixture experiment was performed to try to formulate an SPF40 and SPF70 sunscreen.
- Three components, along with water, were varied. Three components were held fixed (invariant).

Mixture components: 4 (2 to 24)

Total: 77

Units: wt %

	Name	Low	High
A [Mixture]	Avobenzone	1	2
B [Mixture]	Homosalate	5	10
C [Mixture]	Mexoryl SX	1	2
D [Mixture]	Water	63	70

An Example

Sunscreen

- The overall run budget was 30 sunscreen formulations, but only 20 were performed in the first pass of the experiment.

Optimal (Custom) Design

Search: Best Optimality: I

Edit model... Quadratic Scheffe

Blocks: 1 (1 to 1000)

Runs

Required model points: 10

Additional model points: 0

Lack-of-fit points: 5

Replicate points: 5

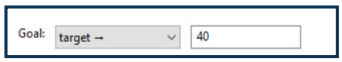
Additional center points: 0

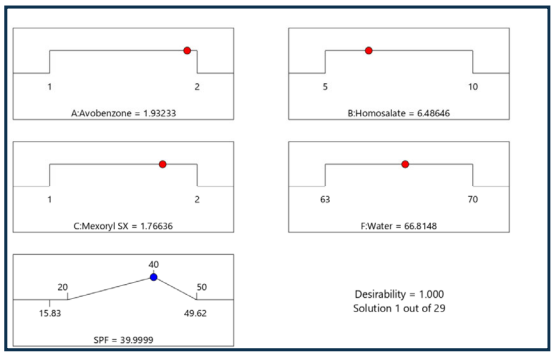
Total runs: 20

An Example

Results

- After the experiment was performed, the model was optimized to find the formulation that achieves an SPF40 formulation:





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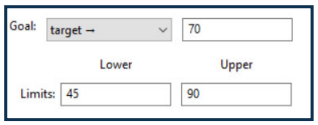
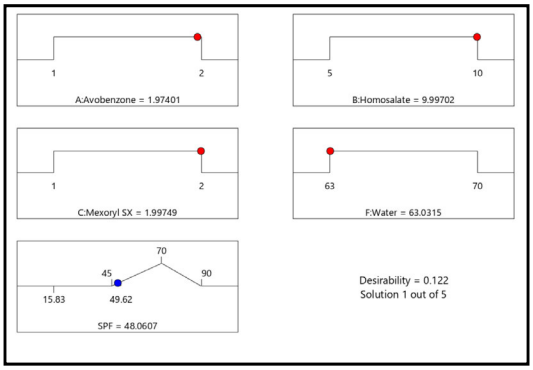
An Example

SPF70?

- Can the data collected under this design produce a sunscreen that's SPF70?





Answer: No!

We can get about as high as SPF48. Clearly some components need to go beyond their original lows/highs.

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An Example

Expand the Space

Design Constraints

Mixture Coding: Actual

Low Limit		Constraint		High Limit
1.000	≤	A:Avobenzone	≤	2.000
5.000	≤	B:Homosalate	≤	10.000
1.000	≤	C:Mexoryl SX	≤	2.000
63.000	≤	D:Water	≤	70.000
		A+B+C+D	=	**77.000**

Edit constraints...

Edit Constraints

Enter constraints in 'Actual' values.

Clear All

Example: 0.05 ≤ 1.4A - 2.5B + C ≤ 0.80

Hide '≤' Columns

Mixture 1 Total: 77.000

Low Limit	≤	Constraint	≤	High Limit
1	≤	+ A	≤	3
5	≤	+ B	≤	15
1	≤	+ C	≤	3
0	≤	+ D	≤	70

OK Cancel Help

Sequential DOE 35

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An Example

Add Runs to the Augmented Space

File Edit View Display Options Design Tools Help

Augment Design... Write Candidate File... Edit Design Model

Augment Add Center Points Replicate Design Blocks

Navigation Pane

Design (Actual) Information Notes

Run	A:Avobenzone wt %		
1	1.5651	5.175	1.53381

Augment Design

Search: Best Optimality: I

Edit model... Quadratic Scheffe

Block: new block

Runs

Required model points: 1

Additional model points: 6

Lack-of-fit points: 0

Replicate points: 0

Additional center points: 0

New runs: 7

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An Example

Block	Run	Component 1 A:Avobenzone wt %	Component 2 B:Homosalate wt %	Component 3 C:Mexoryl SX wt %	Component 4 D:Octisalate wt %	Component 5 E:Oxybenzone wt %	Component 6 F:Water wt %	Component 7 G:Other wt %	Response 1 SPF
Block 1	1	1.31456	10	2	5	6	63.6854	12	44.53
Block 1	2	1.55419	7.8522	1.04961	5	6	66.544	12	24.23
Block 1	3	2	6.91834	2	5	6	66.0817	12	49.62
Block 1	4	1	5	1	5	6	70	12	15.83
Block 1	5	2	10	1.475	5	6	63.525	12	41.72
Block 1	6	2	5	1	5	6	69	12	27.15
Block 1	7	1.97293	5.91388	1.70624	5	6	67.4069	12	37.82
Block 1	8	1	10	1	5	6	65	12	20.72
Block 1	9	1	5	2	5	6	69	12	28.97
Block 1	10	1.55419	7.8522	1.04961	5	6	66.544	12	28.19
Block 1	11	1	8.75002	1.4377	5	6	65.8123	12	29.35
Block 1	12	1	7.74264	1.52547	5	6	66.7319	12	29.71
Block 1	13	1.58486	5	1.54006	5	6	68.8751	12	31.59
Block 1	14	2	10	1.475	5	6	63.525	12	37.34
Block 1	15	1.58486	5	1.54006	5	6	68.8751	12	28.99
Block 1	16	1	7.74264	1.52547	5	6	66.7319	12	27.11
Block 1	17	1.05921	9.23903	2	5	6	64.7018	12	33.04
Block 1	18	1.55419	7.8522	1.04961	5	6	66.544	12	30.12
Block 1	19	1	6.52284	1	5	6	68.4772	12	20.26
Block 1	20	2	9.07158	1.01486	5	6	64.9136	12	26.33
Block 2	21	2	15	1	5	6	59	12	
Block 2	22	3	15	3	5	6	56	12	
Block 2	23	1	15	3			58	12	
Block 2	24	1.5	7.5	1.5			66.5	12	
Block 2	25	3	10	1			63	12	
Block 2	26	3	5	3			66	12	
Block 2	27	1	5	3			68	12	

New high limits are highlighted

Sequential DOE 37

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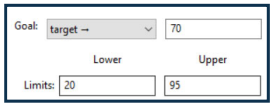
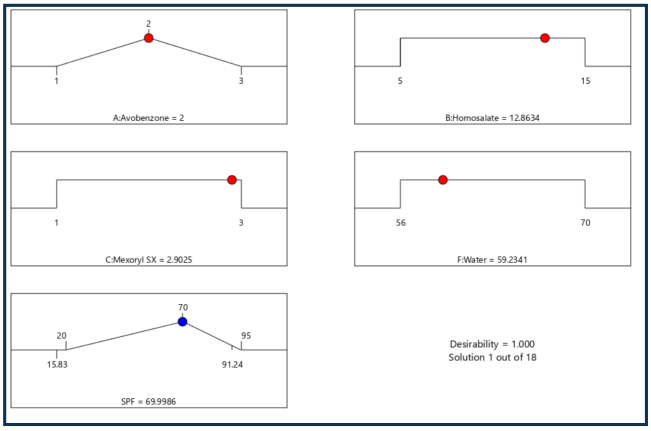
An Example

Re-analyze, Re-Optimize





We can now achieve an SPF70 formulations as well with only 7 additional runs.

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Outline

- Introduction and Motivation
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- An Example
- **Conclusion**

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Follow Up

Benefits of Sequential DOE

- Allows for simultaneous exploration of the design space while maximizing the information that each run provides.
- Avoids situations where you are stuck with suboptimal results and have no budget left for further experimentation.
- Increases precision in regions of high interest.
- In my experience, very little research has been done in this area of DOE, and the few experimenters that are aware of this approach usually do it in an ad-hoc manner.
- Most research on sequential experimentation deals with screening, augmenting to increase the model order, adding/dropping factors.
- **Caveat:** This approach assumes you can change the lows/highs of your factors without great difficulty.

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Conclusion

Take Home Points

- The major point of this presentation is to encourage you to think about DOE as a sequential strategy with regards to the design space, not a one-shot experiment. Even just being aware of this technique can be a huge advantage.
- We generally recommend starting, with 60-75% of your run budget **if possible**. If there are issues with the experiment, the design space can be re-shaped and the remaining runs can be allocated in the new space.
- If there are no issues with the experiment, the remaining runs can be allocated in a smaller region of interest, or back in the original space.
- Design-Expert 13 will allow for factor and mixture component ranges to be changed after a design has been created. It will also include a **Design Space Augment** wizard that guides users through this entire process.

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