REVEALING INTERACTIONS FROM FRACTIONAL DOE'S

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SUMMARY

Highly-fractionated two-level factorials are a powerful tool for making significant improvements to product quality and process efficiency. Unfortunately, this approach to design of experiments (DOE) may alias the main effects with their interactions. Then it is no longer clear which factors truly influence the process. In Part 1, this paper illustrates the use of a graphical technique for viewing alternative aliased interactions. The graphical procedure enhances, but does not remove, the guesswork required when a highly-fractional design produces significant effects. The only sure way to pin down the actual effects will be to perform follow up experiments, which will be discussed in Part 2. A technique called "foldover" is tailor-made for de-aliasing effects. This sequential approach to DOE offers a great deal of flexibility to the quality engineer.

KEY WORDS

Design of Experiments, Aliased Interactions, Fold-over Design

PART I – GRAPHICAL EXPLORATION OF ALIASED INTERACTIONS

Quality engineers are often handed complex problems on a tight time schedule. In the worst case, there's a full-blown quality crisis caused by a process failure or new demands by the customer. The engineer must quickly determine which of many variables must be changed to accomplish a breakthrough improvement. Fractional two-level factorial design of experiments often proves to be the answer. It's particularly useful as a screening tool for identifying significant effects. For example, 7 factors each at 2 levels would require 128 runs in the full factorial design, but by taking fractionation to an extreme, the 7 factors can be studied in as few as 8 runs! However, an 8-run factorial provides only 8 bits of information. This will only be sufficient to estimate the overall mean plus the 7 main effects. Interactions, which occur when the effect of one factor depends on the other, cannot be detected. With 7 factors this requires you assume insignificance for all of the 21 possible two-factor interactions, as well as 99 possible interactions of three or more factors!

Any fractional two-level design, including those proposed by Plackett-Burman and Taguchi, causes effects to be aliased. The true causes for quality problems are then disguised in much the same way that a criminal uses aliased names to escape detection. In the case of 7 factors in 8 runs, each main effect is aliased with 3 two-factor interactions (see below), plus 4 higher order interactions (not shown).

- A = A + BD + CE + FG
- B = B + AD + CF + EG
- C = C + AE + BF + DG
- D = D + AB + CG + EF
- E = E + AC + BG + DF
- F = F + AG + BC + DE
- G = G + AF + BE + CD

The effect that is labeled "A" also represents the BD, CE, and FG interactions. As indicated by the plus signs in the relationship, all the listed effects combine to produce what's listed as "A". It's possible for effects to cancel. Then you miss them entirely. This is a risk you take when doing highly-fractional designs. If anything is significant, you must make an educated guess as to which of the aliased effects is truly active.

It is very helpful at this stage to explore the alternatives graphically. To illustrate the procedure, let's look at a case study from "Statistics for Experimenters" by Box, Hunter and Hunter (page 424). In a newly constructed chemical plant the filtration step takes nearly twice as long as it did at the older plant, resulting in serious process delays. A brainstorming session produces 7 factors thought to affect filtration time (see Table 1.)

After expressing "much disagreement" about some of these "ridiculous" proposals for change, the attendees decide they must do a DOE to pinpoint the actual cause(s) for delays.

Factor	Low Level (–)	High Level (+)
A – water supply	Town	Well
B – raw material	On site	Other
C – temperature	Low	High
D – recycle	Yes	No
E – caustic soda	Fast	Slow
F – filter cloth	New	Old
G – holdup time	Low	High

Table 1: Factors thought to affect filtration time

As shown in Table 2, the seven factors are simultaneously varied from low to high levels over the course of eight experiments. The minus (-) signs and plus (+) signs represent the low and high levels of each factor. This combination of runs produces the alias structure that was described earlier. The runs are shown in standard order but actually were performed in random order.

Std Order	A: Water Source	B: Raw Material	C: Temp	D: Recycle	E: Caustic soda	F: Filter-cloth	G: Holdup Time	Filtration Time
1	–	–	–	+	+	+	–	68.4
2	+	–	–	–	–	+	+	77.7
3	–	+	–	–	+	–	+	66.4
4	+	+	–	+	–	–	–	81.0
5	–	–	+	+	–	–	+	78.6
6	+	–	+	–	+	–	–	41.2
7	–	+	+	–	–	+	–	68.7
8	+	+	+	+	+	+	+	38.7

Table 2: 8 runs performed by experimenters

A half-normal plot of effects (Figure 1) shows that the three largest effects are factors E, C, and A. These three effects stray significantly from the near-zero effects, which fall on a relatively straight line originating from zero. So, at first glance, it appears that factors A-water supply, and C-temperature, and E-caustic soda, should be controlled properly to reduce filtration time.

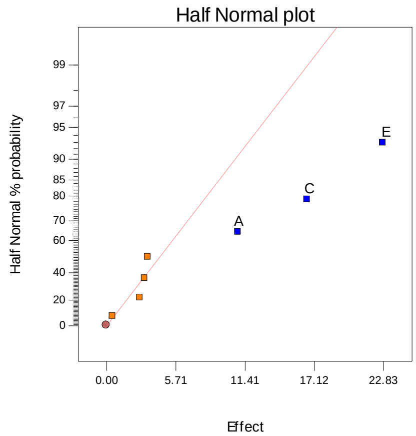

Figure 1: Plot showing effects E, C, and A

Before leaping to that conclusion, remember that A is aliased with the interactions of BD, CE, and FG. Factors C and E are also each aliased with interactions containing the other two effects! There are many possible interactions that could be significant other than the main effects A, C, and E. However, if we assume that the effects exhibit a hierarchical family structure, the following alternatives are most likely:

- A, C, and AC (E is aliased with AC)
- A, E and AE (C is aliased with AE)
- C, E and CE (A is aliased with CE)

Interaction graphs can be created for each of these alternative solutions. Although the interactions are aliased, the graphs look different. An expert in the subject matter may be able to look at these graphs and pick out the interaction that is most likely. The graphs are simple to create by hand. The method will be demonstrated by creating the AC interaction graph. The graph is simply a plot of the average response values at each combination of A and C low and high levels. First, create a table for the A and C level combinations (Table 3). Notice that for this experiment, each combination of A and C occurs twice, so there are two responses. These will be averaged to obtain the point that is plotted on the graph.

A	**C**			**Average**
-	-	68.4	66.4	67.4
-	+	78.6	68.7	73.65
+	-	77.7	81.0	79.35
+	+	41.2	38.7	39.95

Table 3: Responses for A and C

Now, create a graph with A:water on the X-axis and the response values (Filtration Time) on the Y-axis. Factor C:temp does not lie on an axis, but instead it is labeled on the graph. Plot the averages on the graph and draw a line to connect the two points that represent C-minus. Draw another line to connect the two points that represent C-plus. Likewise, interaction graphs can be created for the AE and CE interactions. These are shown in Figure 2.

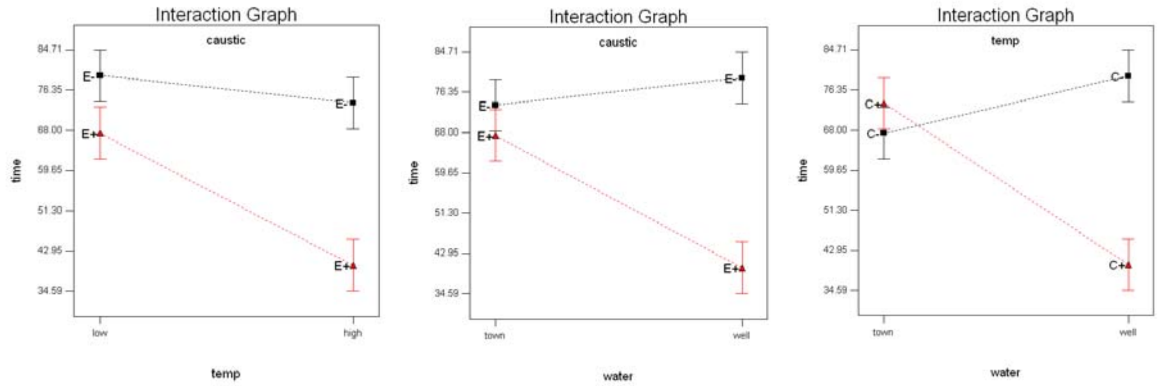

Figure 2: AC Interaction CE Interaction AE Interaction

With the data that has been collected thus far, each of these interaction graph is just as statistically valid as the other. So, how do you know which factors you should control? Are the main effects A, C, and E truly the correct ones? Or do you only need to control two of those factors? If so, which two: A & C, A & E or C & E? Again, a technical expert may be able to use these differing interaction graphs to figure out the answer, but for the rest of us, we'll have to try something else.

PART II – FOLD-OVER TECHNIQUE

We've extracted all the information we can out of 8 runs. More experimentation is needed to resolve the aliased effects. Box, Hunter and Hunter provide a technique called "foldover" to do this most effectively. It involves a systematic change in levels from plus to minus and vice-versa. Which effects are de-aliased depends on whether you fold over some or all the factor columns. In this case, where all main effects are aliased with 2-factor interactions, it will be beneficial to fold over all the factors. By adding a second set of 8 runs that have the opposite signs as the original set (Table 4), you can de-alias the main effects.

Std Order	A: Water Source	B: Raw Material	C: Temp	D: Recycle	E: Caustic soda	F: Filter-cloth	G: Holdup Time	Filtration Time
1	–	–	–	+	+	+	–	68.4
2	+	–	–	–	–	+	+	77.7
3	–	+	–	–	+	–	+	66.4
4	+	+	–	+	–	–	–	81.0
5	–	–	+	+	–	–	+	78.6
6	+	–	+	–	+	–	–	41.2
7	–	+	+	–	–	+	–	68.7
8	+	+	+	+	+	+	+	38.7
9	+	+	+	–	–	–	+	66.7
10	–	+	+	+	+	–	–	65.0
11	+	–	+	+	–	+	–	86.4
12	–	–	+	–	+	+	+	61.9
13	+	+	–	–	+	+	–	47.8
14	–	+	–	+	–	+	+	59.0
15	+	–	–	+	+	–	+	42.6
16	–	–	–	–	–	–	–	67.6

Table 4: Complete Design Layout after Foldover

After you create the foldover, but BEFORE you do the runs, you should always check out the new alias structure to see if the total design will give you the results you expected. Here is the new alias structure for the main effects and two-factor interactions:

- A = A
- B = B
- C = C
- D = D
- E = E
- F = F
- G = G
- AB = AB + CG + EF
- AC = AC + BG + DF
- AD = AD + CF + EG
- AE = AE + BF + DG
- AF = AF + BE + CD
- AG = AG + BC + DE
- BD = BD + CE + FG

The main effects are no longer aliased with 2-factor interactions. However, the 2-factor interactions remain aliased with each other. (Although not shown here, the main effects are actually aliased with 3-factor interactions, but these are generally assumed to be insignificant relative to the others.) Now look at the half-normal probability plot of the effects for the total design (Figures 3). Two effects appear to be significant: the main effect of E and the AE interaction (Figure 4).

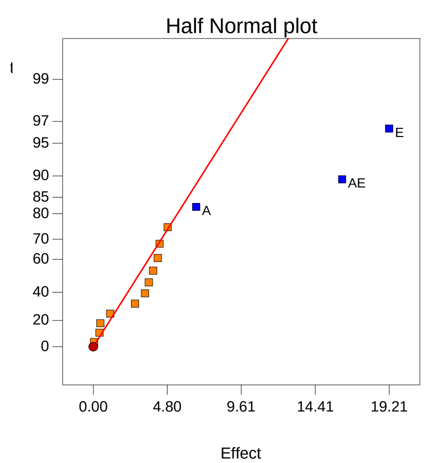

Figure 3: Plot showing effects

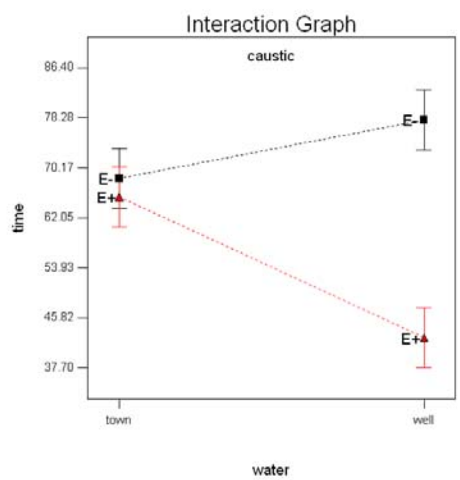

Figure 4: AE Interaction Graph

With the alias structure cleared up, it becomes reasonable to make the assumption that A:water and E:caustic soda are the factors that must be controlled. The experimenters in this plant proved out this theory by performing confirmation runs. Therefore, C:temperature, despite initial indications, is actually not a significant factor. Who knows how much time would’ve been wasted chasing after this non-factor.

CONCLUSION

Small fractional factorial designs are an extremely important tool to use, especially when experiments are expensive and/or time-consuming. However, due to intrinsic aliasing of effects, the conclusions must be drawn carefully. Creating graphs of the aliased interactions can provide valuable information. Better yet, by following up with foldover runs, the experimenter gains control of the aliasing and greatly improves the odds of correctly identifying the true causes for success or failure.

REFERENCES

Box, Hunter, and Hunter, 1978. *Statistics for Experimenters*. John Wiley & Sons.