

This month's webinar presented by:  
Wayne Adams



With the Stat-Ease, Inc.  
consulting team



Pat Whitcomb



Mark Anderson



Shari Kraber

Due to time constraints, please hold questions until the Q&A breaks or after the presentation.

All participants will be muted during the presentation except for breaks. (Not everyone has nice quiet room)

- **Déjà Vu all over again**
  - Define Replicates and Repeats
  - Why use Replicates
    - Brief Power Discussion
  - Replication Example
- **Déjà Vu all over again, again**
  - Why use Repeats
  - Repeat Measures example

➤ **Replicates:**

The same run settings are done again

➤ **Repeats:**

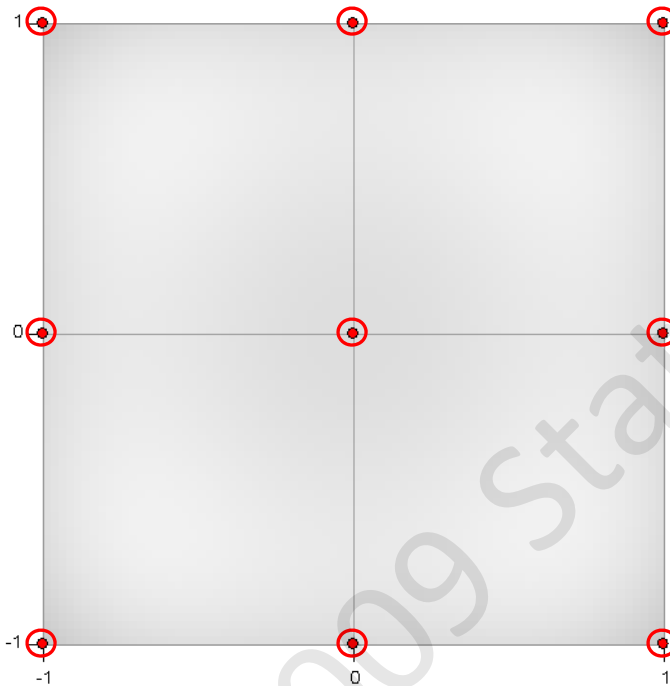
Measurements are done multiple times and averaged into a single response.

### ➤ Design Replicates

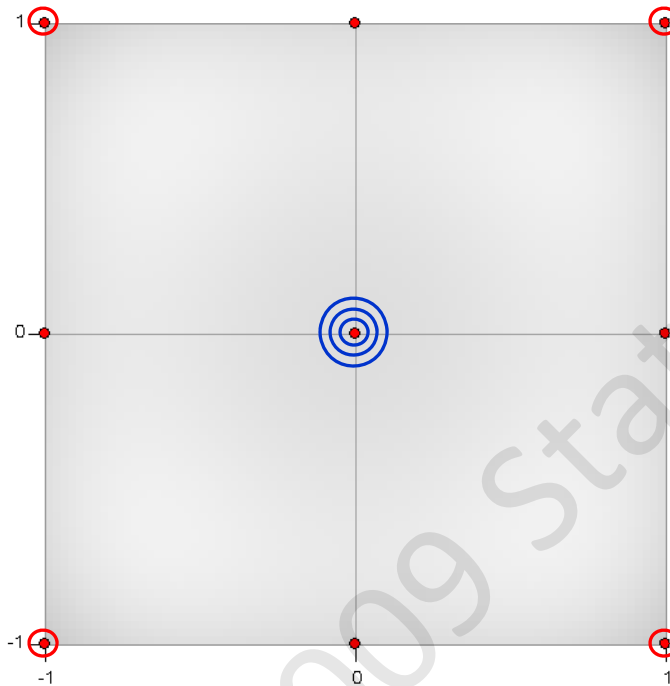
- increase power
- provide an estimate for pure error
- reduce the prediction error

### ➤ Point Replicates

- provide an estimate for pure error
- to a lesser extent increase power
- chosen to reduce over all prediction errors
- are often...
  - center points to detect curvature
  - standard operating conditions



Design Replicates  
multiply the number of  
runs.  
Every run is done again.



Point Replicates are more selective.

Center points are replicated to test for curvature.

Extreme combinations (high leverage) replicates improve prediction errors.

- **A Replicate is a new run.**
- Ideally, the process is reset after each run
  - Calibrating the equipment
  - Cleaning the equipment
  - Changing factor settings, etc.
- All runs should have the same setup process.



Before talking about using replication,  
a quick discussion of power is required

- Explanation of power
- Increasing power
- An example

# What is Power?

## Signal, Noise and all that Greek

$\Delta$  = (Signal) The minimum amount of change to produce a *Eureka* moment

$\sigma$  = (Noise) The expected standard deviation

$\alpha$  = The acceptable risk to find false effects;  
often set to 5%

$\beta$  = The acceptable risk to not find true effects;  
kept as low as possible, 20% or less

Confidence =  $(1-\alpha)*100\%$

Power =  $(1-\beta)*100\%$  -- power to be at least 80%

## The probability of finding an effect!

Power depends on:

- The size of the difference  $\Delta$ :  
the larger the difference the higher the power.
- The size of the experimental error  $\sigma$ :  
the smaller  $\sigma$  the higher the power.
- The  $\alpha$  risk chosen:  
the larger  $\alpha$  the higher the power.
- The design:  
more orthogonal and larger designs have more power.
- The number of replicates:  
replicates give better estimates - increasing power.



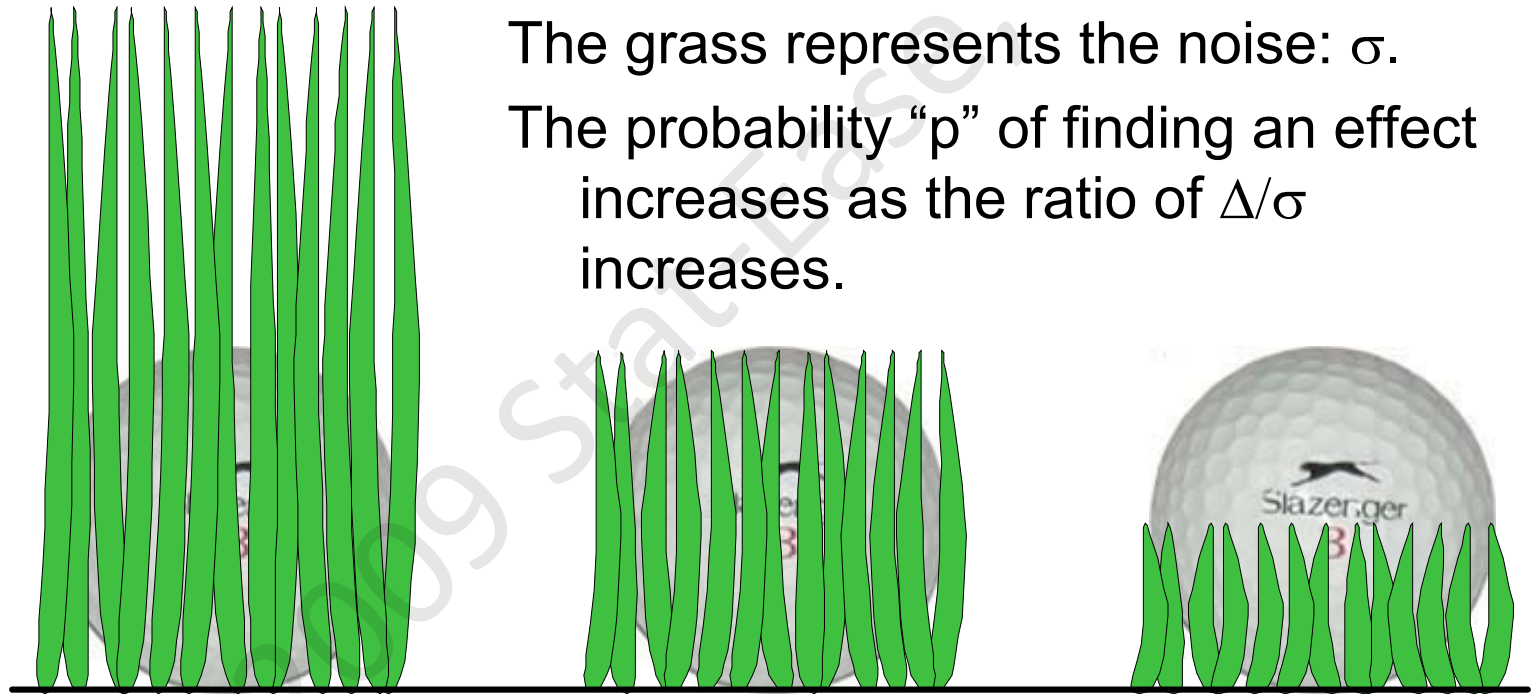
# Power

The probability of finding an effect!

The golf ball represents the signal:  $\Delta$ .

The grass represents the noise:  $\sigma$ .

The probability “p” of finding an effect increases as the ratio of  $\Delta/\sigma$  increases.



$$\Delta/\sigma = 1/2$$
$$p = 8.6\%$$

$$\Delta/\sigma = 1$$
$$p = 19.5\%$$

$$\Delta/\sigma = 2$$
$$p = 57.2\%$$

- Use the Raise-Hand button
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- Explanation of power
- Increasing power
- **A quick example of using replicates**

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


# Replicated Randomized Block Factorial LCD Case Study (*Setup*)

- The objective is to find the best color/typeface combination to maximize readability.
- The experimenters will use a  $2^3$  full factorial design.
- During each of the 8 runs, they will be measuring the seconds to read one of 8, 30 word paragraphs.
- An improvement of 1 second is considered a meaningful improvement in readability.
- Due to the timing relying on people, the expected standard deviation is about 1 second.

1. Identify opportunity and define objective.  
Find the best color/typeface combination to maximize readability - minimize time to read a 30 word paragraph.
2. State objective in terms of measurable responses.  
Response = Time in seconds.
  - a. Define the change ( $\Delta y$ ) that is important to detect for each response.  $\Delta = 1$  second
  - b. Estimate experimental error ( $\sigma$ ) for each response.  $\sigma = 1$  second.
  - c. Use the signal to noise ratio ( $\Delta/\sigma = 1.0$ ) to estimate power.



3. Select the input factors to study. (*Remember that the factor levels chosen determine the size of  $\Delta$ .*)

Factor	Low	High
Foreground color	black 	yellow 
Background color	white 	cyan 
Typeface	arial	TNR*
*Times New Roman		



### 4. Select a design and:

- Evaluate aliases (fractional factorials and/or blocked designs) *Not applicable here.*
- Evaluate power (desire power > 80% for effects of interest) *Order: Main effects.*
- Examine the design layout to ensure all the factor combinations are safe to run and are likely to result in meaningful information (no disasters)

$2^3$  factorial

# Evaluating Power

## LCD Case Study (*Done Once*)

Reading Time:  $\Delta = 1$  sec,  $\sigma = 1$  sec,  $\Delta/\sigma = 1.0$

Power is reported at a 5.0% alpha level to detect the specified signal/noise ratio.			
Recommended power is at least 80%.			
time	sec		
Signal (delta) = 1.00	Noise (sigma) = 1.00		Signal/Noise (delta/sigma) = 1.00
A	B	C	
19.5 %	19.5 %	19.5 %	

**Want power of at least 80% for effects of interest!**

# Replicate to Increase Power

## LCD Case Study (*Two Replicates*)

- Click [**<<Back**] to the factorial design matrix menu and build a  $2^3$  factorial with **2 replicates** (16 runs) and **2 blocks**. (The blocks are **Pat** and **Shari**.) Click [**Continue>>**] to the power evaluation.

Power is reported at a 5.0% alpha level to detect the specified signal/noise ratio.			
Recommended power is at least 80%.			
<b>time</b>	<b>sec</b>		
<b>Signal (delta) = 1.00</b>	<b>Noise (sigma) = 1.00</b>	<b>Signal/Noise (delta/sigma) = 1.00</b>	
A	B	C	
44.6 %	44.6 %	44.6 %	

# Replicate to Increase Power

## LCD Case Study (*Five Replicates*)

- Click [**<<Back**] to the factorial design matrix menu and build a  $2^3$  factorial with **5 replicates** (40 runs) and **5 blocks**. (**Mark**, **Wayne** and **Elicia** are the 3<sup>rd</sup>, 4<sup>th</sup> and 5<sup>th</sup> blocks.) Click [**Continue>>**] to the power evaluation.

Power is reported at a 5.0% alpha level to detect the specified signal/noise ratio.			
Recommended power is at least 80%.			
<b>time</b>	<b>sec</b>		
<b>Signal (delta) = 1.00</b>	<b>Noise (sigma) = 1.00</b>	<b>Signal/Noise (delta/sigma) = 1.00</b>	
A	B	C	
86.6 %	86.6 %	86.6 %	

- Use the Raise-Hand button
- Un-mute your phone if asking the question
- Mute your phone if some one else is asking a question.



- Compare replicates and repeats – again
- Use repeats to improve power and reduce costs
- **A quick example using repeat measurements**

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## Increasing Power by Reducing Variation

**Replication** reduces all sources of variation by a factor of **r**. Where **r** is the number of times the design is completed.

**Repeating Measurements** reduces the variation from only the measurement source by a factor of **m**.  
Where **m** is the number of measurements taken.

Reducing the total variation by any amount increases the signal to noise ratio. Increasing Signal/Noise ratio increases power.



### Question:

If **Repeating Measurements**  $m$  times only decreases measurement variation, why not always **replicate**?

### Answer 1:

It often costs less to re-measure than it does to replicate.

### Answer 2:

If most of the variation is coming from the measurement system – the biggest bang for the buck comes from reducing measurement variation.

**S** = pre-experiment setup costs

S

**C** = cost per run

[C \* c]

**c** = number of runs

**R** = cost per design replicate

[K \* (r-1) \* c]

**r** = number of replicates (iterations)

**M** = cost per measurement

[M \* (m - 1) \* r \* c]

**m** = number of measurements

Total Cost of the Experiment

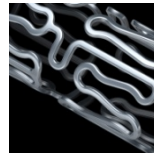
$$= S + [C * c] + [R * (r-1) * c] + [M * (m-1) * r * c]$$

[Spreadsheet for Cost Calculations](#)

- A Repeat is a measurement done again without resetting the system.
  - Repeat measurements are averaged and reported as a single response.
  - Averaged measurements reduce the effect of measurement variability.
  - If the test is destructive, multiple parts are created during each run to be measured later.

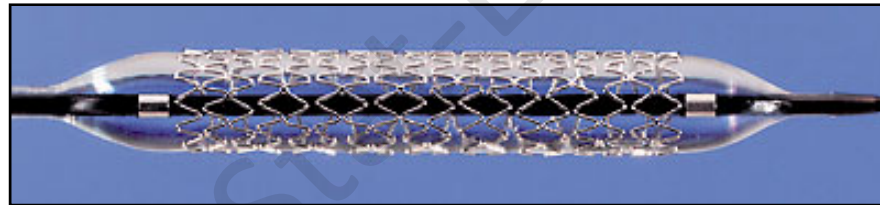
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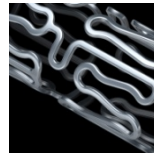


## Repeat Measurements Stent Delivery System

A stent is a wire mesh tube used to prop open an artery that's recently been cleared using angioplasty. The stent is collapsed to a small diameter over a balloon catheter. It's then moved into the area of the blockage.



When the balloon is inflated, the stent expands, locks in place and forms a scaffold. This holds the artery open. The stent stays in the artery permanently, holding it open to improve blood flow to the heart muscle.

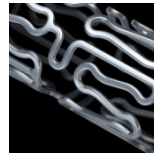


# Repeat Measurements

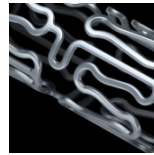
## Stent Delivery System

This case study is meant to illustrate typical DOE use in research to develop an improved product; in this case a stent delivery system. Typical factors include:

- Lengths and diameters of various components, e.g. tip, balloon, catheter, etc.
- Materials used for the components.
- Assembly parameters, e.g. weld locations, how the balloon is folded, etc.
- Stent geometry, wall thickness, how it is crimped on the balloon, etc.



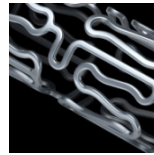
1. Identify opportunity and define objective.  
Relate stent deliverability and safety to process factors.
2. State objective in terms of measurable responses.  
Deliverability is quantified by Trackability and Pushability; safety is quantified by Burst pressure. Want to estimate 2FI model, this requires a res V design.
  - a. Define the change ( $\Delta y$ ) that is important to detect for each response.  $\Delta_{\text{Burst}} = 6$  psig,  $\Delta_{\text{Push}} = 15$  g/cm and  $\Delta_{\text{Track}} = 10$  g\*cm.
  - b. Estimate error ( $\sigma$ ):
    - $\sigma_{\text{Burst}} = 8$  psig;
    - $\sigma_{\text{Push}} = 30$  g/cm;
    - $\sigma_{\text{Track}} = 6$  g\*cm;
  - c. Calculate signal to noise:
    - $\Delta/\sigma = 6/8 = 0.75$
    - $\Delta/\sigma = 15/30 = 0.50$
    - $\Delta/\sigma = 10/6 = 1.67$



3. Select the input factors to study.  
*(The actual factor names and levels are proprietary.)*

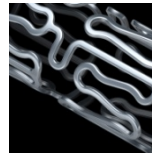
Factor	Type	Low Level (-)	High Level (+)
A	numeric	-1	+1
B	numeric	-1	+1
C	numeric	-1	+1
D	numeric	-1	+1
E	numeric	-1	+1
F	numeric	-1	+1
G	numeric	-1	+1
H	numeric	-1	+1
J	numeric	-1	+1
K	numeric	-1	+1
L	categorical	L1	L2





### 4a Select a design:

- Evaluate aliases (fractional factorials and/or blocked designs) *During build*
- Evaluate power (desire power > 80% for effects of interest) *Order: Main effects*
- Examine the design layout to ensure all the factor combinations are safe to run and are likely to result in meaningful information (no disasters)



# MR5 Design: Power Stent Delivery System

Power is reported at a 5.0% alpha level to detect the specified signal/noise ratio.  
Recommended power is at least 80%.

## Burst psig

**Signal (delta) = 6.00 Noise (sigma) = 8.00 Signal/Noise (delta/sigma) = 0.75**

A	B	C	D	E	F	G	H	J	K	L
85.6 %	85.6 %	85.7 %	85.6 %	85.5 %	85.3 %	85.9 %	86.0 %	85.9 %	85.8 %	88.9 %

## Push g/cm

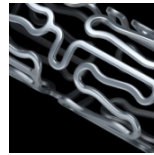
**Signal (delta) = 15.00 Noise (sigma) = 30.00 Signal/Noise (delta/sigma) = 0.50**

A	B	C	D	E	F	G	H	J	K	L
52.2 %	52.3 %	52.4 %	52.3 %	52.1 %	51.9 %	52.5 %	52.7 %	52.5 %	52.4 %	56.4 %

## Track g\*cm

**Signal (delta) = 10.00 Noise (sigma) = 6.00 Signal/Noise (delta/sigma) = 1.67**

A	B	C	D	E	F	G	H	J	K	L
99.9 %	99.9 %	99.9 %	99.9 %	99.9 %	99.9 %	99.9 %	99.9 %	99.9 %	99.9 %	99.9 %

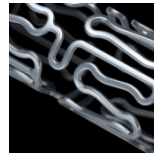


## MR5 Design: Power Stent Delivery System

Power is low (~52%) for Push; to increase power:

1. Increase design size: replicating the design 76x2=152 runs gives adequate power (~82%), but there are too many runs to be practical.
2. Increase  $\Delta_{\text{Push}} = 15$  g/cm: No – we are interested in a difference of 15 g/cm.
3. Decrease  $\sigma_{\text{Push}} = 30$  g/cm: By partitioning the variance, we determine that the push measurement contributes most (75%) of the variation. Repeating the measurement (not the experimental run) to reduce  $\sigma$  is the answer.

[See the Stent Numbers.](#)



# MR5 Design: Power Stent Delivery System

$$\sigma_{\text{Push}} = 30 \quad \& \quad \sigma_{\text{Push}}^2 = \sigma_{\text{Process}}^2 + \sigma_{\text{Measurement}}^2$$

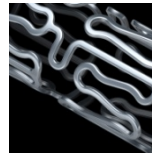
$$900 = 225 + 675 \quad \therefore \quad \% \text{Variation} = 675/900 = 75\%$$

Make three independent push measurements for each run.  
Enter the average of the measurements as the response:

Then by the CLT  $\left( \sigma_{\text{Average}}^2 = \frac{\sigma_{\text{Measurement}}^2}{m} \right)$ :

$$\sigma_{\text{Push}}^2 = 225 + \frac{675}{3} = 450 \quad \% \text{Variation} = 50\%$$

$$\sigma_{\text{Push}} = \sqrt{450} \approx 21 \quad \frac{\Delta}{\sigma} = \frac{15}{21} = 0.71$$



# MR5 Design: Power Stent Delivery System

Power is reported at a 5.0% alpha level to detect the specified signal/noise ratio.  
Recommended power is at least 80%.

## Burst psig

**Signal (delta) = 6.00 Noise (sigma) = 8.00 Signal/Noise (delta/sigma) = 0.75**

A	B	C	D	E	F	G	H	J	K	L
85.6 %	85.6 %	85.7 %	85.6 %	85.5 %	85.3 %	85.9 %	86.0 %	85.9 %	85.8 %	88.9 %

## Push g/cm

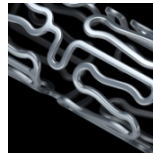
**Signal (delta) = 15.00 Noise (sigma) = 30.00 Signal/Noise (delta/sigma) = 0.50**

A	B	C	D	E	F	G	H	J	K	L
82.1 %	82.1 %	82.2 %	82.1 %	82.0 %	81.8 %	82.4 %	82.5 %	82.4 %	82.3 %	85.8 %

## Track g\*cm

**Signal (delta) = 10.00 Noise (sigma) = 6.00 Signal/Noise (delta/sigma) = 1.67**

A	B	C	D	E	F	G	H	J	K	L
99.9 %	99.9 %	99.9 %	99.9 %	99.9 %	99.9 %	99.9 %	99.9 %	99.9 %	99.9 %	99.9 %



## Summary:

- Replicating runs will reduce the system error; from both process and measurement.
- Repeating the measurement reduces only the measurement error.
- The magnitude of each of these errors and the relative cost of replicating runs versus repeating measurements dictates which will give the most “bang” for your buck.

# Increasing Power

## It's all about the Power

As shown in the LCD Case Study, power was increased by **replication** (adding more runs).

In the Stent Case Study **repeating measurements** proved to be a more cost effective method to reduce the noise variation.

The two techniques can be combined to reduce costs and variation.

# Increasing Power

## Or Maybe its About Reducing Variation

**Replication** reduces the total variation by a factor of  $r$  the number of times the design is completed.

$$\sigma_{total}^2 = \frac{\sigma_{process}^2}{r} + \frac{\sigma_{measurement}^2}{m \cdot r},$$

where  $k$  is the number of design iterations and  
 $m$  is the number of measurements

Reducing the total variation increases the signal to noise ratio, thus increasing power.



# Increasing Power Or Maybe Just Some of the Variation

**Repeating Measurements**  $m$  times only reduces the variation from the measurement source.

$$\sigma_{total}^2 = \frac{\sigma_{process}^2}{r} + \frac{\sigma_{measurement}^2}{m \cdot r},$$

where  $k$  is the number of design iterations and  
 $m$  is the number of measurements

Unless the measurement variation makes up a large percentage of the total, repeating measurements may not help very much.

Feel free to download the presentation and data file(s) used for today's webinar.

## Webinar Downloads

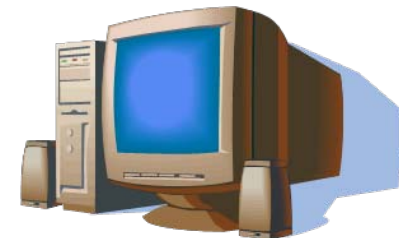
<http://www.statease.com/webinar.html>

You should find

- one PDF for the presentation

- one Excel File

- two Design-Expert® files



# How to get help



- ❑ Search publications posted at [www.stateease.com](http://www.stateease.com)
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- ❑ E-mail [stathelp@stateease.com](mailto:stathelp@stateease.com) for answers from Stat-Ease's staff of statistical consultants
- ❑ Call 612.378.9449 and ask for "statistical help"

*Thanks for attending!*